PHYSQ 126 – Quiz 2 (26 janvier 2016) Solutions

Flow Velocity of Blood Conceptual Question

Arteriosclerotic plaques forming on the inner walls of arteries can decrease the effective cross-sectional area of an artery. Even small changes in the effective area of an artery can lead to very large changes in the blood pressure in the artery and possibly to the collapse of the blood vessel. Imagine a healthy artery, with blood flow velocity of v_0=0.14 m/s and mass per unit volume of ρ = 1050 kg/m^3. The kinetic energy per unit volume of blood is given by

$$K_0 = \frac{1}{2}\rho v_0^2 .$$

Imagine that plaque has narrowed an artery to one-fifth of its normal cross-sectional area (an 80% blockage).

A. Compared to normal blood flow velocity, v_0, what is the velocity of blood as it passes through this blockage?

Hints (1)

Hint 1. Continuitity equation and reduced cross-sectional area

By the equation of continuity, as the cross-sectional area of an artery decreases because of plaque formation, the velocity of blood through that region of the artery will increase. The new flow speed can be calculated by rearranging the equation of continuity,

$$A_1 v_1 = A_2 v_2;$$

so

$$v_2 = \frac{A_1}{A_2} v_1$$

where A_1 and A_2 are the initial and final cross-sectional areas, and v_1 and v_2 are the initial and final velocities of the blood, respectively.

ANSWER:

- ○ $80 v_0$
- ○ $20 v_0$
- ⊙ $5 v_0$
- ○ $v_0/5$

B. By what factor does the kinetic energy per unit of blood volume change as the blood passes through this blockage?

$K_0 = \frac{1}{2}\rho v_0^2 \propto v_0^2$ gives

ANSWER:

- (selected) 25
- 5
- 1

C. As the blood passes through this blockage, what happens to the blood pressure?

Hints (2)

Hint 1. Blood pressure and blood velocity

Bernoulli's equation states that the sum of the pressure, the kinetic energy per volume, and the gravitational energy per volume of a fluid is constant. For initial and final pressures p_1 and p_2, initial and final velocities v_1 and v_2, and mass per unit volume of blood, ρ, ignoring the effects of changes in gravitational energy leads to

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2.$$

Basically, the sum of kinetic energy and pressure must remain constant in an artery. This leads to a very serious health risk. As blood velocity increases, blood pressure in a section of artery can drop to a dangerously low level, and the blood vessel can collapse, completely cutting off blood flow, owing to lack of sufficient internal pressure.

Hint 2. Calculating the change in blood pressure

From Bernoulli's equation, the change in pressure is the negative of the change in kinetic energy per unit volume. For initial and final kinetic energies per unit volume of the blood, K_1 and K_2, respectively,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2,$$

or

$$p_1 + K_1 = p_2 + K_2,$$

where $K = (1/2)\rho v^2$. Rearranging this equation yields

$$p_2 - p_1 = -(K_2 - K_1)$$

or

$$\Delta p = -\Delta K.$$

or $\Delta P = P_2 - P_1 = -\frac{\rho}{2}\left(v_2^2 - v_1^2\right) = -\frac{1050}{2}\left((5\times 0.14)^2 - (0.14)^2\right) = -247$ Pa

ANSWER:

- ○ It increases by about 250 Pa.
- ○ It increases by about 41 Pa.
- ○ It stays the same.
- ○ It decreases by about 41 Pa.
- ◉ It decreases by about 250 Pa.

Since the kinetic energy increases by a factor of 25,

$$\Delta K = 25 \times K_0 - K_0 = 24 \times K_0 = 247 \text{ Pa} \approx 250 \text{ Pa}.$$

Bernoulli's equation tells you that $\Delta K = -\Delta p$.

As the blood velocity increases through a blockage, the blood pressure in that section of the artery can drop to a dangerously low level. In extreme cases, the blood vessel can collapse, completely cutting off blood flow, owing to lack of sufficient internal pressure. In the next three parts, you will see how a small increase in blockage can cause a much larger pressure change.

For parts D - F imagine that plaque has grown to a 90% blockage. **same as above**

D. Relative to its initial, healthy state, by what factor does the velocity of blood increase as the blood passes through this blockage?

10

E. By what factor does the kinetic energy per unit of blood volume increase as the blood passes through this blockage?

100

F. What is the magnitude of the drop in blood pressure, Δp, as the blood passes through this blockage? Use K_0 as the normal (i.e., unblocked) kinetic energy per unit volume of the blood.

$\Delta p = 1000 \text{ Pa}$