

Kirchoff's Rules and Applying Them

Description: Questions elicit the physics behind Kirchoff's rules. Then gives example and suggestions on how to use them.

Learning Goal:

To understand the origins of both of Kirchoff's rules and how to use them to solve a circuit problem.

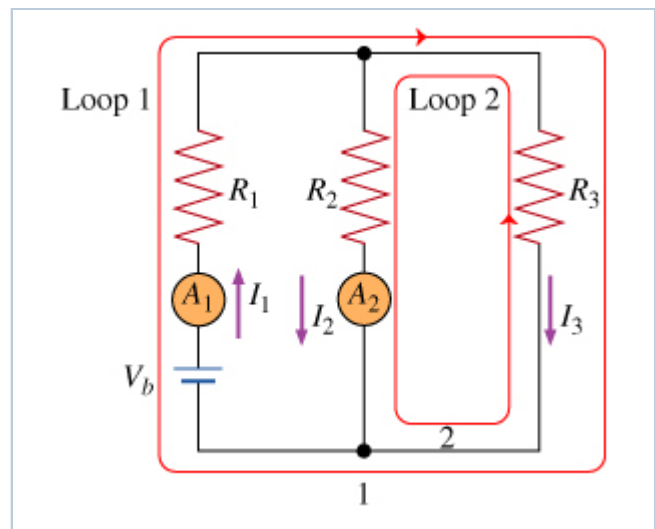
This problem introduces Kirchoff's two rules for circuits:

- *Kirchoff's loop rule:* The sum of the voltage changes across the circuit elements forming any closed loop is zero.
- *Kirchoff's junction rule:* The algebraic sum of the currents into (or out of) any junction in the circuit is zero.

The figure shows a circuit that illustrates the concept of *loops*, which are colored red and labeled loop 1 and loop 2. Loop 1 is the loop around the entire circuit, whereas loop 2 is the smaller loop on the right. To apply the loop rule you would add the voltage changes of all circuit elements around the chosen loop. The figure contains two junctions (where three or more wires meet)--they are at the ends of the resistor labeled R_3 . The battery supplies a constant voltage V_b , and the resistors are labeled with their resistances. The ammeters are ideal meters that read I_1 and I_2 respectively.

The direction of each loop and the direction of each current arrow that you draw on your own circuits are arbitrary. Just assign voltage drops consistently and sum both voltage drops and currents algebraically and you will get correct equations. If the actual current is in the opposite direction from your current arrow, your answer for that current will be negative. The direction of any loop is even less important:

The equation obtained from a counterclockwise loop is the same as that from a clockwise loop except for a negative sign in front of every term (i.e., an inconsequential change in overall sign of the equation because it equals zero).



Part A

The junction rule describes the conservation of which quantity? Note that this rule applies only to circuits that are in a steady state.

[Hints](#) (1)

Hint 1. At the junction

Think of the analogy with water flow. If a certain current of water comes to a split in the pipe, what can you say (mathematically) about the sum of the three water currents at this junction? If this were not true, water would accumulate at the junction.

ANSWER:

- current
- voltage
- resistance

Part B

Apply the junction rule to the junction labeled with the number 1 (at the bottom of the resistor of resistance R_2).

Answer in terms of given quantities, together with the meter readings I_1 and I_2 and the current I_3 .

[Hints](#) (1)

Hint 1. Elements in series

The current through resistance R_1 is not labeled. You should recognize that the current I_1 passing through the ammeter also passes through resistance R_1 because there is no junction in between the resistor and the ammeter that could allow it to go elsewhere. Similarly, the current passing through the battery must be I_1 also. Circuit elements connected in a string like this are said to be *in series* and the same current must pass through each element. This fact greatly reduces the number of independent current values in any practical circuit.

ANSWER:

$$\Sigma I = 0 = I_3 + I_2 - I_1$$

Also accepted: $-I_3 - I_2 + I_1$

If you apply the junction rule to the junction above R_2 , you should find that the expression you get is equivalent to what you just obtained for the junction labeled 1. Obviously the conservation of charge or current flow enforces the same relationship among the currents when they separate as when they recombine.

Part C

Apply the loop rule to loop 2 (the smaller loop on the right). Sum the voltage changes across each circuit element around this loop going in the direction of the arrow. Remember that the current meter is ideal.

Express the voltage drops in terms of V_b , I_2 , I_3 , the given resistances, and any other given quantities.

[Hints](#) (3)

Hint 1. Elements in series have same current

The current through the ammeter is I_1 , and this current has to go through the resistor of resistance R_1 because there is no junction in between that could add or subtract current. Similarly, the current passing through the battery must be I_1 also. Circuit elements connected in a string like this are said to be *in series* and the same current must pass through each element. This fact greatly reduces the number of independent current values in any practical circuit.

Hint 2. Sign of voltage across resistors

In determining the signs, note that if your chosen loop traverses a particular resistor in the *same* direction as the current through that resistor, then the end it enters through will have a more positive potential than the end from which it exits by the amount IR . Thus the voltage change across that resistor will be negative. Conversely, if your chosen loop traverses the resistor in the opposite direction from its current arrow, the voltage changes across the resistor will be positive. Let these conventions govern your equations (i.e., don't try to figure out the direction of current flow when using the Kirchhoff loop--decide when you put the current arrows on the resistors and stick with that choice).

Hint 3. Voltage drop across ammeter

An ideal ammeter has zero resistance. Hence there is no voltage drop across it.

ANSWER:

$$\Sigma(\Delta V) = 0 = I_3 R_3 - I_2 R_2$$

Also accepted: $-I_3 R_3 + I_2 R_2$

Part D

Now apply the loop rule to loop 1 (the larger loop spanning the entire circuit). Sum the voltage changes across each circuit element around this loop going in the direction of the arrow.

Express the voltage drops in terms of V_b , I_1 , I_3 , the given resistances, and any other given quantities.

ANSWER:

$$\Sigma(\Delta V) = 0 = V_b - I_1 R_1 - I_3 R_3$$

Also accepted: $-(V_b - I_1 R_1 - I_3 R_3)$

There is one more loop in this circuit, the inner loop through the battery, both ammeters, and resistors R_1 and R_2 . If you apply Kirchhoff's loop rule to this additional loop, you will generate an extra equation that is redundant with the other two. In general, you can get enough equations to solve a circuit by either

1. selecting all of the internal loops (loops with no circuit elements inside the loop) or
2. using a number of loops (not necessarily internal) equal to the number of internal loops, with the extra proviso that at least one loop pass through each circuit element.