Faraday's Law and Induced Emf

Description: Discusses Faraday's law; presents a sequence of questions related to finding the induced emf under different circumstances.

Learning Goal:

To understand the terms in Faraday's law and to be able to identify the magnitude and direction of induced emf.

Faraday's law states that induced emf is directly proportional to the time rate of change of magnetic flux. Mathematically, it can be written as

$$\mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t},$$

where $\mathcal{E}$ is the emf induced in a closed loop, and

$$\frac{\Delta \Phi_B}{\Delta t}$$

is the rate of change of the magnetic flux through a surface bounded by the loop. For uniform magnetic fields the magnetic flux is given by $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos(\theta)$, where θ is the angle between the magnetic field $\vec{B}$ and the normal to the surface of area A.

To find the *direction* of the induced emf, one can use Lenz's law:

The induced current's magnetic field opposes the change in the magnetic flux that induced the current.

For example, if the magnetic flux through a loop *increases*, the induced magnetic field is directed *opposite* to the "parent" magnetic field, thus countering the increase in flux. If the flux *decreases*, the induced current's magnetic field has the same direction as the parent magnetic field, thus countering the decrease in flux.

Recall that to relate the direction of the electric current and its magnetic field, you can use the *right-hand rule*: When the fingers on your right hand are curled in the direction of the current in a loop, your thumb gives the direction of the magnetic field generated by this current.

In this problem, we will consider a rectangular loop of wire with sides x and y placed in a region where a uniform magnetic field $\vec{B}$ exists as shown in . The resistance of the loop is R. For the purposes of this problem we will assume that the area vector $\vec{A}$ points out of the page.

Initially, the field is perpendicular to the plane of the loop and is directed out of the page. The loop can rotate about either the vertical or horizontal axis, passing through the midpoints of the opposite sides, as shown.
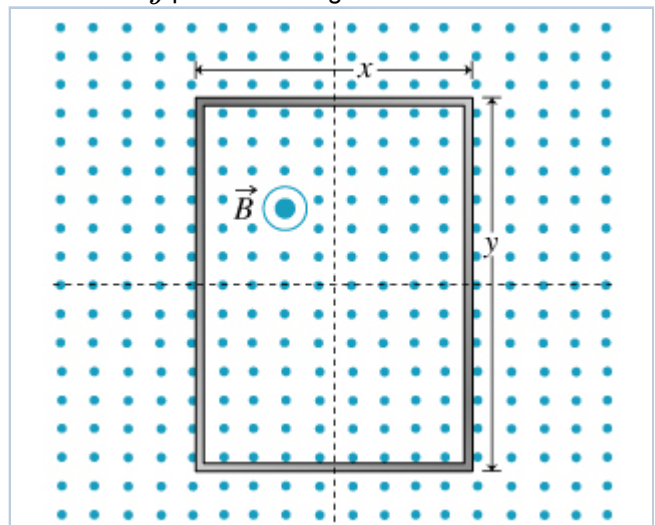
Part A

Which of the following changes would induce an electromotive force (emf) in the loop? When you consider each option, assume that no other changes occur.

Check all that apply.

ANSWER:

- ☑ The magnitude of $\vec{B}$ increases.
- ☑ The magnitude of $\vec{B}$ decreases.
- ☑ The loop rotates about the vertical axis (vertical dotted line) shown in the diagram.
- ☑ The loop rotates about the horizontal axis (horizontal dotted line) shown in the diagram.
- ☐ The loop moves to the right while remaining in the plane of the page.
- ☐ The loop moves toward you, out of the page, while remaining parallel to itself.

Part B

Find the flux Φ_B through the loop.

Express your answer in terms of x, y, and B.

ANSWER:

$$\Phi_B = \; xyB$$

Part C

If the magnetic field steadily decreases from B to zero during a time interval t, what is the magnitude $\mathcal{E}$ of the induced emf?

Express your answer in terms of x, y, B, and t.

Hints (1)

ANSWER:

$$\mathcal{E} = \frac{xyB}{t}$$

Part D

If the magnetic field steadily decreases from B to zero during a time interval t, what is the magnitude I of the induced current?

Express your answer in terms of x, y, B, t, and the resistance R of the wire.

ANSWER:

$$I = \frac{xyB}{Rt}$$

Part E

If the magnetic field steadily decreases from B to zero during a time interval t, what is the direction of the induced current?

ANSWER:

- ○ clockwise
- ⦿ counterclockwise

> The flux decreases, so the induced magnetic field must be in the same direction as the original (parent) magnetic field. Therefore, the induced magnetic field is out of the page. Using the right-hand rule, we deduce that the direction of the current is counterclockwise.

Part F

Which of the following changes would result in a clockwise emf in the loop? When you consider each option, assume that no other changes occur.

Check all that apply.

ANSWER:

- ☑ The magnitude of $\vec{B}$ increases.
- ☐ The magnitude of $\vec{B}$ decreases.
- ☐ The loop rotates through 45 degrees about the vertical axis (vertical dotted line) shown in the diagram.
- ☐ The loop rotates through 45 degrees about the horizontal axis (horizontal dotted line) shown in the diagram.
- ☐ The loop moves to the right while remaining in the plane of the page.
- ☐ The loop moves toward you, out of the page, while remaining parallel to itself.

> Clockwise emf implies that the induced magnetic field is directed into the page. Therefore, the magnetic flux of the original field must be increasing. Only the first option corresponds to increasing flux.