Part A: multiple choice questions

1. B
   $F_A = m(g+a)$, $F_B = F_C = m(g+a)/2$

2. B

3. E

4. D

5. D

6. B
   Work done by conservative force is path-independent and dot($\mathbf{F}$, $\mathbf{r}_2 - \mathbf{r}_1$) = 12 J.

7. A
   Particle has constant speed in all cases, so only need to compare power of weight. Powers of weight in (A) and (E) both have the same magnitude, but sign in (E) is opposite to the one in the question.

8. C
   Speed is independent of mass, so kinetic energy is proportional to mass.

9. C
   Direction of linear momentum has changed.

10. D
    Impulse of the force is $2\times10^6\times(10)^3\,\mathbf{i} + 1.6\times10^6\times(10)^4\,\mathbf{j} = 2\times10^9\,\mathbf{i} + 1.6\times10^{10}\mathbf{j}$, so the change in velocity is $2\times10^3\,\mathbf{i} + 1.6\times10^4\mathbf{j}$. The final velocity is therefore $5000\,\mathbf{i} + 12000\,\mathbf{j}$ (m/s), and speed is 13 km/s.

11. A

12. A
    Impulse from 0 to 9s is +18, and from 9 to 12s is –9, so need another –9 to get zero velocity. 9/6=1.5s and 12+1.5 = 13.5s.

13. E
    Total impulse from 0 to 9s is 18, and from 0 to 15s is –9, so maximum speed occurs at 9s.

14. C
    At 9s, speed is 18/2 = 9 m/s.

15. B
    Total linear momentum is zero. Net linear momentum of A and B is to the left, so cart is moving to the right.

16. A
17. C
    Linear momentum conservation: 3a=5b, 4a+12b=5.6, so a=0.5 and b=0.3. Speed of A is 5a=2.5 m/s; speed of B is 13b=3.9 m/s.

18. C
The coefficient of restitution is only related to vertical component of the velocities before and after impact, so only *h* and *d* are needed.

19. A
Torque is zero when **r** and **F** are parallel.

20. E
Use right hand rule for cross product (**k** × (–**i**) = – **j**).

21. C
Energy is lost when the kinetic energy decreases, or angular speed decreases, which occurs in intervals 3 and 6.

22. A
Work done is equal to change in kinetic energy, which is: 2 (interval 1), 0 (interval 2), –2 (interval 3), 0.5 (interval 4), 0 (interval 5), –0.5 (interval 6).

23. B
Rolling without slipping: $\omega = v_{cm}/R$. v_{cm} is the same for front and rear wheels, so ω is inversely proportional to radius. 16 × (15/8) = 30 rpm.

24. C
Before the rope is shortened, $I = Md^2/2$, and kinetic energy is $Md^2\omega_0^2/4$. After the rope is shortened, $I = Md^2/8$, angular momentum conservation gives the new angular speed as $4\omega_0$, so kinetic energy is $Md^2\omega_0^2$. Change is $3Md^2\omega_0^2/4$.

Part B: long-answer questions

Question 1:

a) LM conservation for A+B during collision: $m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$

$$\Rightarrow \quad 8v_{A1} = 8v_{A2} + 2(4.8) \qquad \text{or} \qquad v_{A1} = v_{A2} + 1.2 \qquad (1.1)$$

Coefficient of restitution: $e = \dfrac{v_{B2} - v_{A2}}{v_{A1} - v_{B1}}$

$$\Rightarrow \quad 0.5 = \frac{4.8 - v_{A2}}{v_{A1}} \qquad \text{or} \qquad 0.5v_{A1} = 4.8 - v_{A2} \qquad (1.2)$$

Adding (1.1) and (1.2) yields: $v_{A1} = \dfrac{4.8+1.2}{1.5} = 4 \text{ m/s}$

Energy conservation for sphere A during its swing: $m_A g l (1 - \cos\alpha) = \dfrac{1}{2} m_A v_{A1}^2$

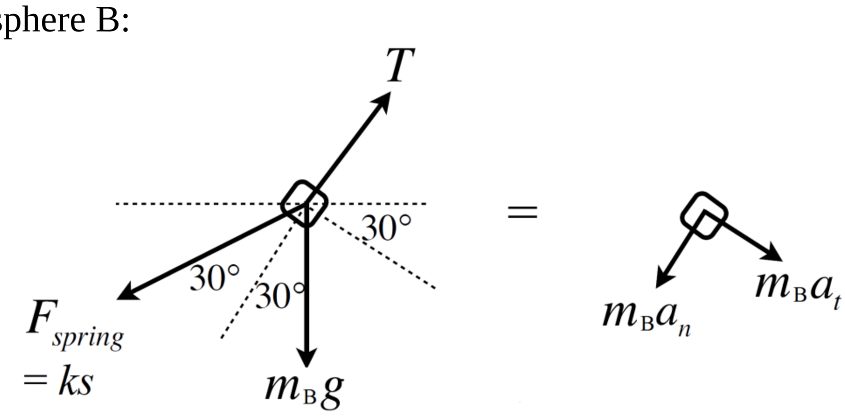
$$\Rightarrow \quad 10(1.6)(1-\cos\alpha) = \frac{1}{2}(4^2) \quad \Rightarrow \quad \cos\alpha = \frac{1}{2} \text{ and } \quad \alpha = 60^o$$

b) Stretch in the spring is $s = \sqrt{3}L - \sqrt{2}L = (\sqrt{3} - \sqrt{2})(2.6) = 0.826 \text{ m}$

Energy conservation for sphere B: $\dfrac{1}{2} m_B v_{B2}^2 + m_B g L(1-\cos\theta) = \dfrac{1}{2} m_B v_{B3}^2 + \dfrac{1}{2} k s^2$

$$\Rightarrow \frac{2(4.8)^2}{2} + 2(10)(2.6)(1-\cos 30^o) = \frac{2v_{B3}^2}{2} + \frac{52(0.826)^2}{2} \Rightarrow v_{B3} = 3.50 \text{ m/s}$$

FBD/KD of sphere B:



In the normal direction: $m_B g \cos 30^o + ks \cos 30^o - T = m_B a_n = m_B \dfrac{v_{B3}^2}{L}$

$$\Rightarrow \quad T = 2(10)\cos 30^o + 52(0.826)\cos 30^o - 2\left(\frac{3.50^2}{2.6}\right) = 45.1 \text{ N}$$

Question 2:

Define position of block A (s_A) downward from the lowest pulley.

Then: $s_A + 2x = \text{constant} \quad \Rightarrow \quad v_A = -2v_B$

As block A falls through a height of 2 m: $\Delta s_A = 2\text{ m}$ and $\Delta x = -1\text{ m}$.

Principle of work and energy for A and B:

$$m_A g \Delta s_A + \mu_k m_B g \Delta x + \int_{1.5}^{0.5} F(x)dx = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2, \text{ so}$$

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B\left(\frac{v_A}{2}\right)^2 = m_A g \Delta s_A + \mu_k m_B g \Delta x + \int_{1.5}^{0.5}\left[2K(x_e - x) + 4Q(x_e - x)^3\right]dx$$

$$= m_A g \Delta s_A + \mu_k m_B g \Delta x + \left[-K(x_e - x)^2 - Q(x_e - x)^4\right]_{1.5}^{0.5}$$

Substitute in numbers:

$$\frac{1}{2}\left(4 + \frac{8}{4}\right)v_A^2 = 4(10)(2) + 0.5(8)(10)(-1) - 6\left(2^2 - 1^2\right) - 0.4\left(2^4 - 1^4\right)$$

$$\Rightarrow \quad v_A = 2.31 \quad \text{m/s}.$$

Question 3:

a) Let A = pulley and B = sphere, energy conservation:

$$Mgx\sin\theta = \frac{1}{2}kx^2 + \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}Mv_{B,cm}^2 + \frac{1}{2}I_B\omega_B^2$$

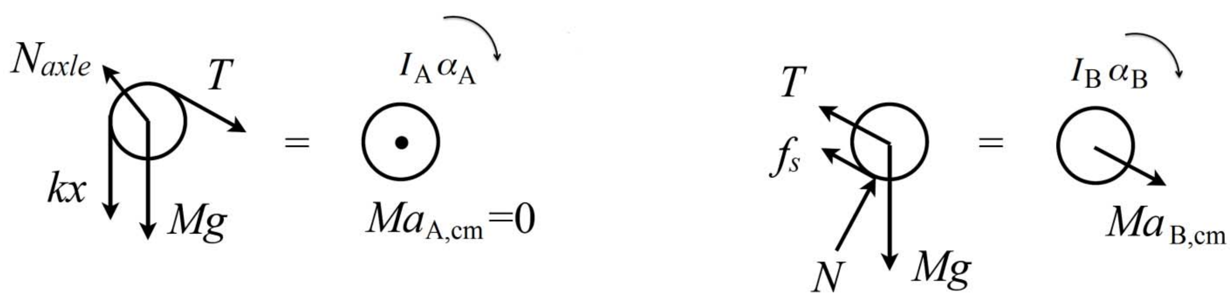
No-slip conditions for the cylinder and for the sphere: $\omega_A = \frac{v_{B,cm}}{R}, \quad \omega_B = \frac{v_{B,cm}}{R}$

$$\Rightarrow \quad Mgx\sin\theta = \frac{1}{2}kx^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{B,cm}}{R}\right)^2 + \frac{1}{2}Mv_{B,cm}^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{B,cm}}{R}\right)^2$$

$$\Rightarrow \quad 20Mgx\sin\theta = 10kx^2 + 19Mv_{B,cm}^2$$

$$\Rightarrow \quad v_{B,cm} = \sqrt{\frac{20}{19}gx\sin\theta - \frac{10k}{19M}x^2}$$

b) FBD/KD for A and B:



No-slip conditions for the cylinder and for the sphere: $\alpha_A = \frac{a_{B,cm}}{R}, \quad \alpha_B = \frac{a_{B,cm}}{R}$

For A:

CW "+" $\quad \Rightarrow \quad (T - kx)R = \left(\frac{1}{2}MR^2\right)\frac{a_{B,cm}}{R} \quad$ or $\quad T - kx = \frac{1}{2}Ma_{B,cm}$ (3.1)

For B:

Down the incline "+" $\quad \Rightarrow \quad Mg\sin\theta - f_s - T = Ma_{B,cm}$ (3.2)

CW "+" $\quad \Rightarrow \quad f_sR = \left(\frac{2}{5}MR^2\right)\frac{a_{B,cm}}{R} \quad$ or $\quad Ma_{B,cm} = \frac{5}{2}f_s$ (3.3)

Adding (3.1) and (3.2) and substituting in (3.3) $\quad \Rightarrow \quad f_s = \frac{4}{19}\left(Mg\sin\theta - kx\right)$