**Faculty of Engineering and Department of Physics**
**ENPH 131 Final Examination**
**Monday, April 28, 2014; 2:00 pm – 4:30 pm**
**Universiade Pavilion**

**Section EB01 (Kaminsky): Rows 1, 3, 5 (seats 1-45)**
**Section EB02 (Beach): Rows 5 (seats 46-50), 7, 9, 11 (seats 1-35)**
**Section EB03 (McDonald): Rows 11 (seats 36-50), 13, 15, 17 (seats 1-10)**
**Section EB04 (Tang): Rows 17 (seats 11-50), 19, 21**
**Section EB05 (Wheelock): Rows 23, 25, 27 (seats 1-30)**
**Section EB06 (Ropchan): Rows 27 (seats 31-50), 29, 31, 33 (seats 1-20)**

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**There are two separate parts to this exam. This is PART A and the instruction is given below. Please refer to the front page of Part B for its instruction.**

1. **You must "bubble in" your name, student ID number, and class section number on the General Purpose Answer Sheet provided, or your exam will not be marked.** Do this FIRST.
   - Enter your name under "NAME", starting from the Surname.
   - Enter your student ID number under "IDENTIFICATION NUMBER" beginning in the first column. Your student ID number is the 7-digit number on your One Card.
   - Enter your class section number under "SPECIAL CODES". For example if your section number is EB01 then enter 111 in the first three columns of "SPECIAL CODES".
2. Part A is composed of **24 multiple choice questions**. Each question has only one correct answer and is worth 1 point.
3. Answer for each question must be entered on the General Purpose Answer Sheet. This handout (multiple choice questions and formula sheet) will NOT be collected or marked.
4. Formula sheet is included at the end of Part A. You may detach them.
5. No notes or textbooks allowed. Non-programmable calculator approved by Faculty of Engineering allowed. Turn off all unauthorized electronic devices (cell-phones, laptops, tablets, watches with internet/storage capability, etc.), put them in your backpack and place the backpack behind or underneath your chair.
6. **The General Purpose Answer Sheet will be collected at 4:00 pm. Place the completed General Purpose Answer Sheet face down on the upper right corner of your desk.**

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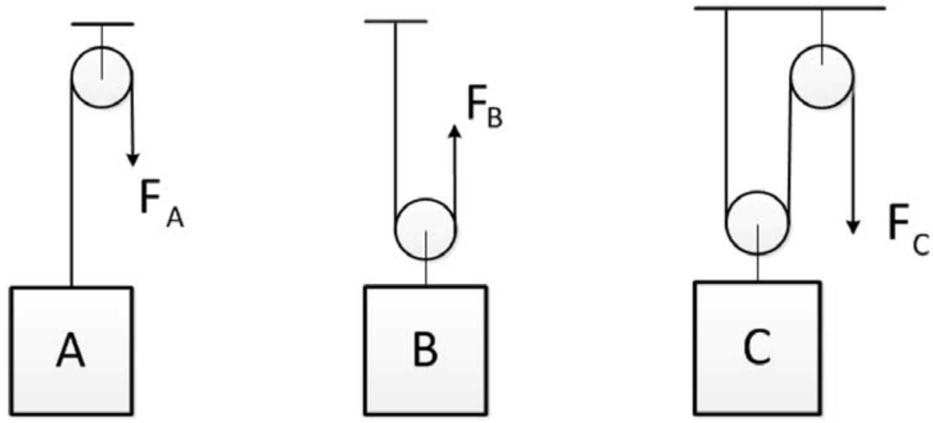
1. Blocks A, B, and C, each of mass $m$, are connected to the massless pulley systems shown below. A force is applied at the end of each rope to lift each block with the same upward acceleration of magnitude $a$.




Rank the magnitude of the force ($F_A$, $F_B$, and $F_C$), from greatest to least noting any ties with an equal sign.

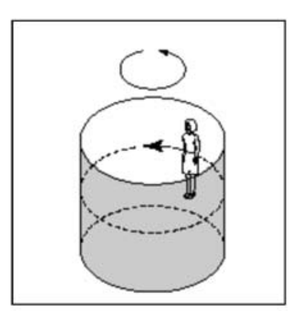
(A) $F_A > F_B > F_C$
(B) $F_A > F_B = F_C$
(C) $F_A = F_B > F_C$
(D) $F_B > F_A > F_C$
(E) $F_A > F_C > F_B$

For Questions 2 to 5, identify the physical force (if applicable) that gives rise to the normal and tangential components of the acceleration in each scenario. Neglect air resistance in all cases.

(A) Gravity
(B) Normal force
(C) Kinetic friction
(D) Static friction
(E) Not applicable

<u>Scenario 1</u>

In a 'barrel of fun' carnival ride the riders stand against a vertical cushion on the inside of a cylinder as shown. After the barrel is 'spun up' to a **constant final angular speed**, the floor of the ride is removed and the passengers do not fall.



2. $a_n$ is due to: A B C D E

3. $a_t$ is due to: A B C D E

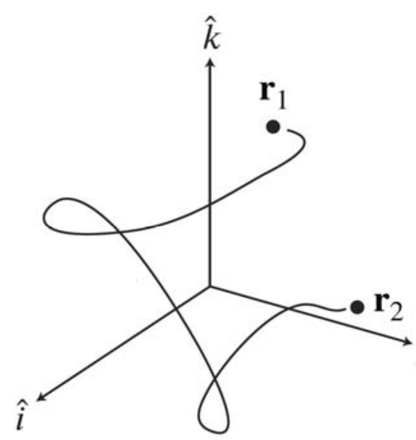
Scenario 2

A coin rests on a horizontal turntable (a disk that can rotate about a vertical axis passing through its center). The turntable undergoes a **constant angular acceleration** and the coin remains at rest relative to the turntable.

4. $a_n$ is due to: A B C D E

5. $a_t$ is due to: A B C D E

6. As a particle moves from $\mathbf{r}_1 = (\mathbf{j} + 4\mathbf{k})$ m to $\mathbf{r}_2 = (\mathbf{i} + 4\mathbf{j} + \mathbf{k})$ m along the curved path shown on the right, one of the forces acting on it is a **conservative** force $\mathbf{F} = (9\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$ N. The work done by $\mathbf{F}$ on the particle as it moves from $\mathbf{r}_1$ to $\mathbf{r}_2$ is
(A) 0 J
(B) 12 J
(C) 16 J
(D) 28 J
(E) impossible to compute without an explicit expression for the path




7. You push a block of mass $m$ up a frictionless ramp of incline $0 < \theta < 30^\circ$ at constant speed $v$. In which of the following scenarios are you providing exactly the same power?
(A) Pushing a block of mass $m/2$ up an incline $\theta$ at constant speed $2v$
(B) Pushing a block of mass $m$ up an incline $2\theta$ at constant speed $v/2$
(C) Pushing a block of mass $2m$ up an incline $\theta/2$ at constant speed $v$
(D) Pushing a block of mass $2m$ up an incline $2\theta$ at constant speed $v/2$
(E) Pushing a block of mass $m$ down an incline $\theta$ at constant speed $v$

8. Objects A and B, with masses $m_A$ and $m_B = 3m_A$, are dropped from a tall building at the same time. If air resistance can be ignored, how do the kinetic energies of A and B compare in the instant before impact on the ground?
(A) The kinetic energies are the same.
(B) The kinetic energy of A is three times that of B.
(C) The kinetic energy of A is one third that of B.
(D) The kinetic energy of A is nine times that of B.
(E) The kinetic energy of A is one ninth that of B.

9. A car is traveling **south** at 50 km/hr and later is traveling **northwest** at 50 km/hr. Which of the following statements is true for the car?
(A) Its kinetic energy and linear momentum both remain unchanged.
(B) Its kinetic energy changes but linear momentum remains unchanged.
(C) Its kinetic energy remains unchanged but its linear momentum changes.
(D) It kinetic energy and linear momentum both change.
(E) Insufficient information to answer.

10. A rocket of mass $1\times10^6$ kg is traveling in deep space at a velocity

$$\mathbf{v} = 3000\,\mathbf{i} - 4000\,\mathbf{j}\ \text{(m/s)}$$

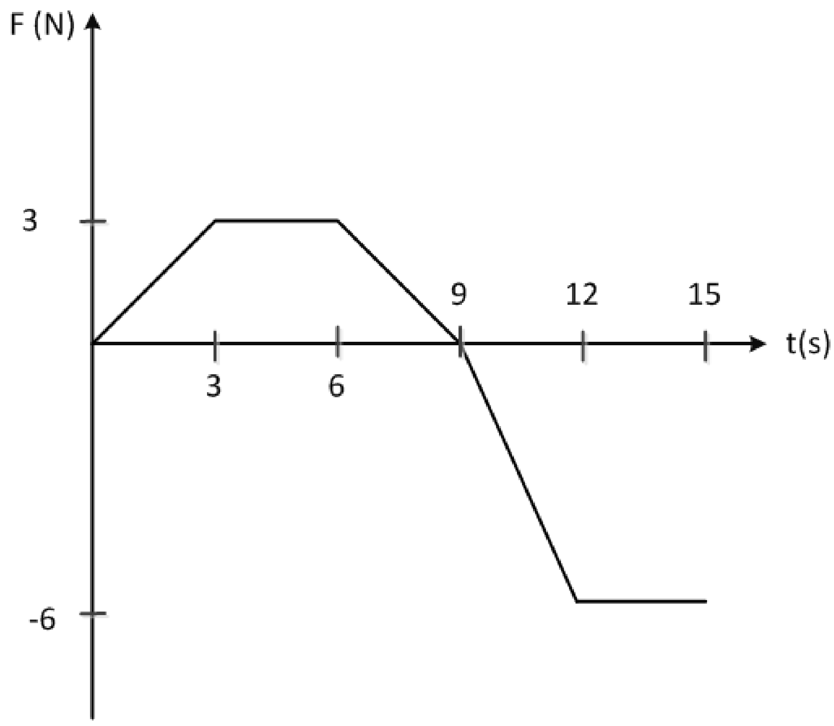
as measured in some inertial reference frame. The thrusters are fired for 10 seconds which exert a time-dependent reaction force on the rocket given by

$$\mathbf{F} = 6\times10^6\,t^2\,\mathbf{i} + 6.4\times10^6\,t^3\,\mathbf{j}\ \text{(N)},$$

where $t$ is in second. Neglecting mass changes of the rocket, what is the speed of the rocket after 10 seconds (as seen by the same inertial observer)?
(A) 6 km/s
(B) 8 km/s
(C) 10 km/s
(D) 13 km/s
(E) 17 km/s

11. A chef accidentally drops a glass on a floor. On which type of floor is the glass more likely to break on impact: Concrete or softwood? Why?
(A) The concrete floor due to short collision time with large impact force.
(B) The concrete floor due to large collision time with large impact force.
(C) The softwood floor due to large collision time with large impact force.
(D) The softwood floor due to large collision time with small impact force.
(E) Neither because the impulse could be the same.

Questions 12 to 14 are based on the diagram below. A horizontal force F(*t*) is applied to a 2 kg box on a frictionless horizontal surface between time *t* = 0 s and *t* = 15 s. At time *t* = 0 s the box is stationary.

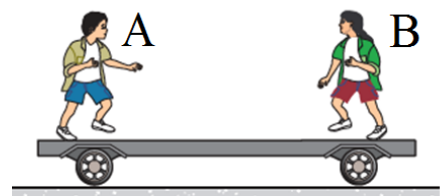



12. At what time *t* does the direction of the box's motion change?
(A) 13.5 s
(B) 9 s
(C) 12.5 s
(D) 6 s
(E) 10.5 s

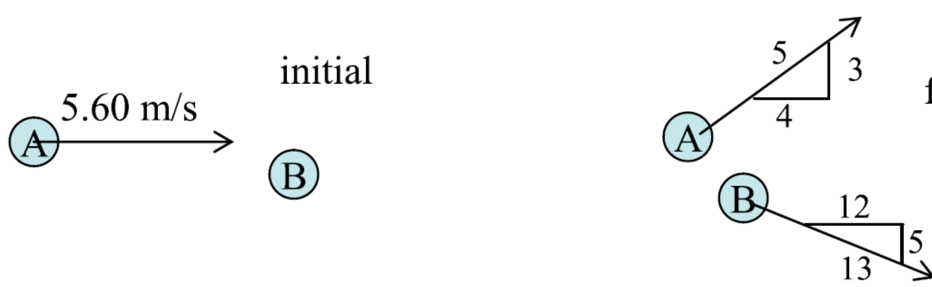
13. At what time does the box achieve its maximum speed?
(A) 6 s
(B) 15 s
(C) 12 s
(D) 3 s
(E) 9 s

14. What is the maximum speed the box achieves?
(A) 18 m/s
(B) 4.5 m/s
(C) 9 m/s
(D) 36 m/s
(E) 3 m/s

15. Two children of equal weight stand on a cart; both children and the cart are at rest. The children then walk toward each other. Child A walks with a constant speed of 1.0 m/s and B walks with a constant speed of 1.5 m/s. Neglecting rolling friction, what will happen to the cart? The cart will:
    (A) roll to the left
    (B) roll to the right
    (C) first roll to the left and then roll to the right
    (D) first roll to the right and then roll to the left
    (E) remain stationary



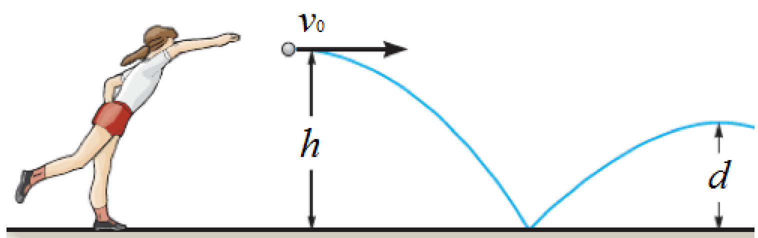

Questions 16 to 17 are based on the diagram below. Hockey puck A is shot along the x-axis at 5.60 m/s at **identical** hockey puck B which is initially at rest on horizontal frictionless ice. The collision is off-center, and they scatter at the angles shown.




16. What is the speed $v_{A2}$ after the collision?
    (A) 2.5 m/s
    (B) 3.2 m/s
    (C) 3.9 m/s
    (D) 4.5 m/s
    (E) 5.0 m/s

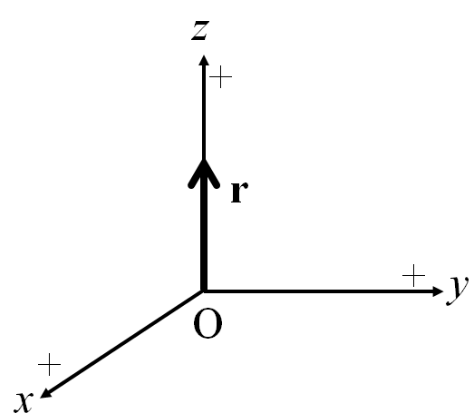
17. What is the speed $v_{B2}$ after the collision?
    (A) 2.5 m/s
    (B) 3.2 m/s
    (C) 3.9 m/s
    (D) 4.5 m/s
    (E) 5.0 m/s

18. A child throws a ball horizontally from an initial height of $h$ with an initial speed of $v_0$. After bouncing off the floor, the ball reaches a height of $d$. Neglect air resistance. What values are necessary in order to completely determine the coefficient of restitution between the ball and the floor?




(A) $v_0$ and $h$, but not $d$
(B) $v_0$ and $d$, but not $h$
(C) $h$ and $d$, but not $v_0$
(D) All three values $v_0$, $h$ and $d$ are needed
(E) Need more information than the three values

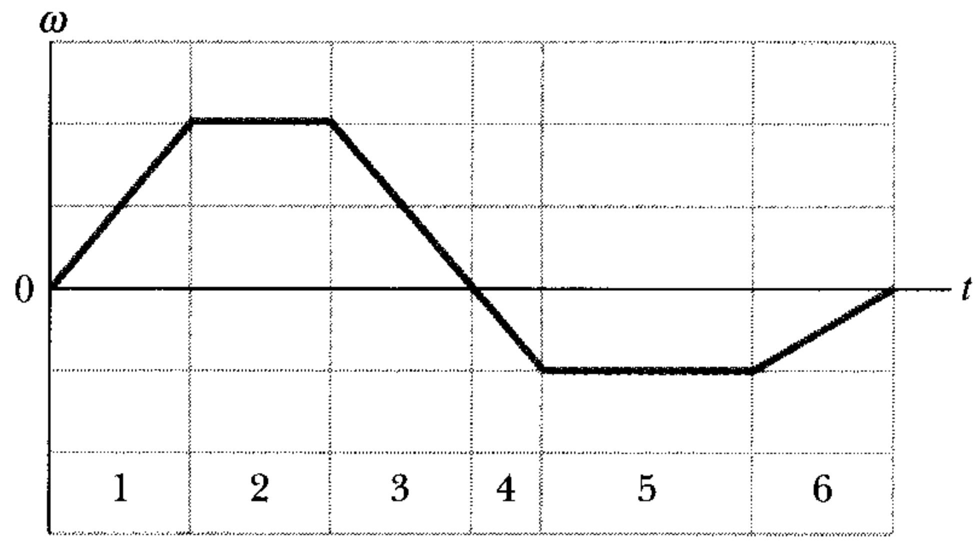
This problem statement applies to Questions 19 to 20. The position vector **r** (|**r**|>0) of a particle points along the positive direction of the $z$-axis. A force **F** is applied on the particle, which can only act in the coordinate directions ($x$, $y$ or $z$). The torque (or moment) $\mathbf{M}_o$ of this force about the origin is given for two scenarios listed below. For each case, specify the direction of the force.




19. If $\mathbf{M}_o = \mathbf{0}$, then the force **F** is
(A) along the $z$-axis
(B) along the positive $y$-axis
(C) along the negative $y$-axis
(D) along the positive $x$-axis
(E) along the negative $x$-axis

20. If $\mathbf{M}_o$ is along the negative $y$-axis, then the force **F** is
(A) along the $z$-axis
(B) along the positive $y$-axis
(C) along the negative $y$-axis
(D) along the positive $x$-axis
(E) along the negative $x$-axis

Questions 21 to 22 are based on the graph below, which shows the angular velocity $\omega$ vs. time $t$ of a merry-go-round being acted upon by an external force. Time intervals 1, 2, 3 and 6 are twice as long as interval 4, and interval 5 is three times as long as interval 4. The vertical axis is drawn to scale.

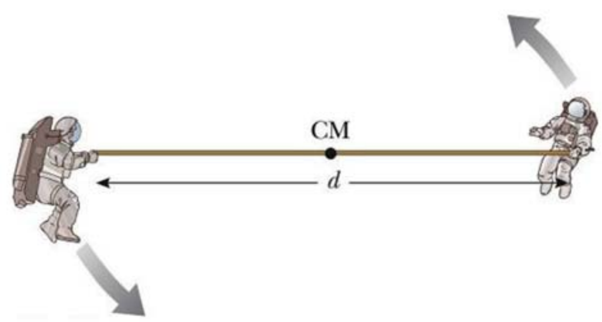



21. During which interval(s) is energy lost by the merry-go-round?
    (A) 1 and 6
    (B) 3 and 4
    (C) 3 and 6
    (D) 4 and 6
    (E) 2 and 5

22. Rank the time intervals according to the work done on the merry-go-round by the applied force, from greatest to least noting any ties with an equal sign.
    (A) 1>4>2=5>6>3
    (B) 1=3>2=4=5>6
    (C) 2=5>3>6>1>4
    (D) 1=3>4=6>2=5
    (E) None of the above.

23. A child is riding a tricycle along a straight horizontal road. The pedals are attached directly to the front wheel, which has a radius of 15 cm. The rear wheels are smaller and have a radius of 8 cm. There is no slip between the wheels and the road surface. If the child is pedaling at 16 rpm (revolutions per minute), then the angular speed of the rear wheels is closest to:
(A) 40 rpm
(B) 30 rpm
(C) 24 rpm
(D) 20 rpm
(E) 15 rpm

24. Two astronauts, each having a mass *M*, are connected by a rope of length *d* having negligible mass. They are isolated in space, orbiting their center of mass at an angular speed of $\omega_0$. By pulling on the rope, the astronauts shorten the distance between them to *d*/2. Treating the astronauts as point particles, what is the **change in kinetic energy** of the system? (Hint: apply angular momentum conservation first.)
(A) $Md^2\omega_0^2 / 4$
(B) $Md^2\omega_0^2 / 2$
(C) $3Md^2\omega_0^2 / 4$
(D) $Md^2\omega_0^2$
(E) $7Md^2\omega_0^2 / 4$



## Fundamental Equations of Dynamics

### KINEMATICS

**Particle Rectilinear Motion**

| *Variable a* | *Constant* $a = a_c$ |
|---|---|
| $a = \dfrac{dv}{dt}$ | $v = v_0 + a_c t$ |
| $v = \dfrac{ds}{dt}$ | $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$ |
| $a\,ds = v\,dv$ | $v^2 = v_0^2 + 2a_c(s - s_0)$ |

**Particle Curvilinear Motion**

| *x, y, z Coordinates* | | *r, θ, z Coordinates* | |
|---|---|---|---|
| $v_x = \dot{x}$ | $a_x = \ddot{x}$ | $v_r = \dot{r}$ | $a_r = \ddot{r} - r\dot{\theta}^2$ |
| $v_y = \dot{y}$ | $a_y = \ddot{y}$ | $v_\theta = r\dot{\theta}$ | $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ |
| $v_z = \dot{z}$ | $a_z = \ddot{z}$ | $v_z = \dot{z}$ | $a_z = \ddot{z}$ |

*n, t, b Coordinates*

$v = \dot{s}$ $\quad a_t = \dot{v} = v\dfrac{dv}{ds}$

$a_n = \dfrac{v^2}{\rho} \quad \rho = \dfrac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

*Relative Motion*

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

**Rigid Body Motion About a Fixed Axis**

| *Variable* $\alpha$ | *Constant* $\alpha = \alpha_c$ |
|---|---|
| $\alpha = \dfrac{d\omega}{dt}$ | $\omega = \omega_0 + \alpha_c t$ |
| $\omega = \dfrac{d\theta}{dt}$ | $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$ |
| $\omega\,d\omega = \alpha\,d\theta$ | $\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$ |

*For Point P*

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

**Relative General Plane Motion—Translating Axes**

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

**Relative General Plane Motion—Trans. and Rot. Axis**

$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

### KINETICS

**Mass Moment of Inertia** $I = \int r^2\,dm$

*Parallel-Axis Theorem* $I = I_G + md^2$

*Radius of Gyration* $k = \sqrt{\dfrac{I}{m}}$

**Equations of Motion**

| | |
|---|---|
| *Particle* | $\Sigma\mathbf{F} = m\mathbf{a}$ |
| *Rigid Body (Plane Motion)* | $\Sigma F_x = m(a_G)_x$<br>$\Sigma F_y = m(a_G)_y$<br>$\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$ |

**Principle of Work and Energy**

$T_1 + \Sigma U_{1-2} = T_2$

***Kinetic Energy***

| | |
|---|---|
| *Particle* | $T = \frac{1}{2}mv^2$ |
| *Rigid Body (Plane Motion)* | $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ |

***Work***

| | |
|---|---|
| *Variable force* | $U_F = \int F\cos\theta\,ds$ |
| *Constant force* | $U_F = (F_c\cos\theta)\,\Delta s$ |
| *Weight* | $U_W = -W\,\Delta y$ |
| *Spring* | $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ |
| *Couple moment* | $U_M = M\,\Delta\theta$ |

**Power and Efficiency**

$P = \dfrac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \quad \varepsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

**Conservation of Energy Theorem**

$T_1 + V_1 = T_2 + V_2$

***Potential Energy***

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

**Principle of Linear Impulse and Momentum**

| | |
|---|---|
| *Particle* | $m\mathbf{v}_1 + \Sigma\int \mathbf{F}\,dt = m\mathbf{v}_2$ |
| *Rigid Body* | $m(\mathbf{v}_G)_1 + \Sigma\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$ |

**Conservation of Linear Momentum**

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

**Coefficient of Restitution** $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

**Principle of Angular Impulse and Momentum**

| | |
|---|---|
| *Particle* | $(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$<br>where $H_O = (d)(mv)$ |
| *Rigid Body (Plane motion)* | $(\mathbf{H}_G)_1 + \Sigma\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2$<br>where $H_G = I_G\omega$<br>$(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$<br>where $H_O = I_O\omega$ |

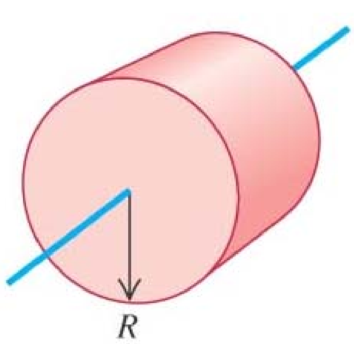
**Conservation of Angular Momentum**

$\Sigma(\text{syst. }\mathbf{H})_1 = \Sigma(\text{syst. }\mathbf{H})_2$

## Mass Moment of Inertia of Homogeneous Solids

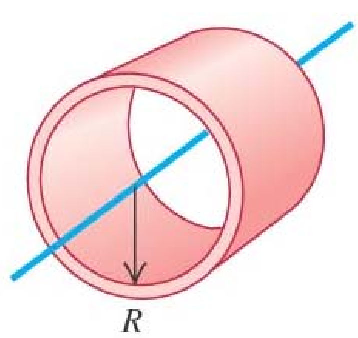
Solid cylinder

$I = \dfrac{1}{2}MR^2$



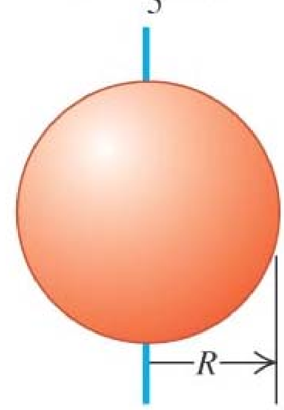

Thin-walled hollow cylinder

$I = MR^2$




Solid sphere

$I = \dfrac{2}{5}MR^2$



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**There are two separate parts to this exam. This is PART B and the instruction is given below. Please refer to the front page of Part A for its instruction.**

1. **Write your name and student ID number below and circle your section number.**
2. Part B has **3 problems** and is out of **36 points**. The value of each question is indicated in the table below. Budget your time accordingly. Attempt all parts of all problems.
3. Write your solution directly on the pages with questions. Indicate clearly if you use the backs of pages for material to be marked. Do not separate the pages containing problems. Show all work in a neat and logical manner. Specify your final numerical answers with 3 significant figures unless otherwise specified.
4. Formula sheets are included at the end of Part A. You may detach them.
5. No notes or textbooks allowed. Non-programmable calculator approved by Faculty of Engineering allowed. Turn off all unauthorized electronic devices (cell-phones, laptops, tablets, watches with internet/storage capability, etc.), put them in your backpack and place the backpack behind or underneath your chair.
6. **If you do not submit your exam prior to 4:20 pm, please remain seated till we finish collecting ALL the exams from the entire class.**

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Last Name: ______________________________

First Name: ______________________________

ID#: ______________________________

Please circle the name of your instructor:

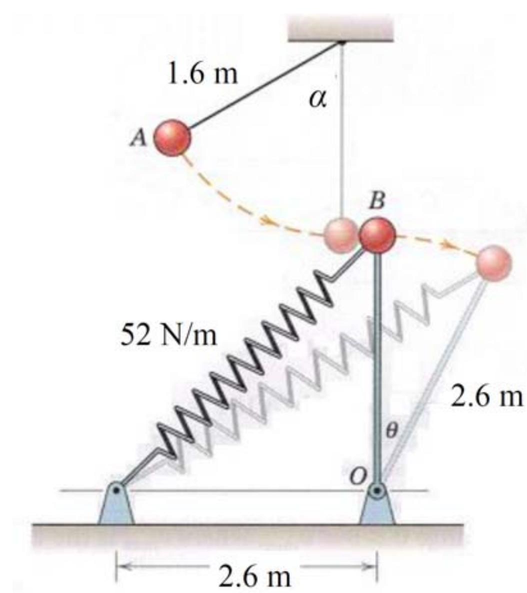
**EB01:** Kaminsky **EB02:** Beach **EB03:** McDonald

**EB04:** Tang **EB05:** Wheelock **EB06:** Ropchan

**Please do not write in the table below.**

| **Question** | **Value (Points)** | **Mark** |
|---|---|---|
| **Part A** | **24** | |
| **Part B, Problem 1** | **15** | |
| **Part B, Problem 2** | **7** | |
| **Part B, Problem 3** | **14** | |
| **Total** | **60** | |

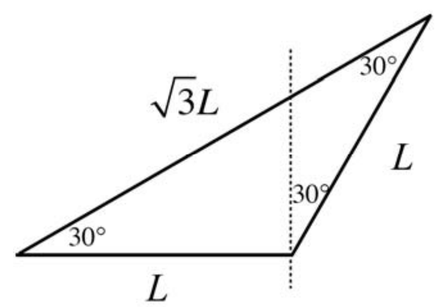
1. [15 marks] An **8.0 kg** sphere A attached to a rope is held at an angle $\alpha$ with respect to the vertical as shown and released from rest. At the bottom of its swing, sphere A strikes the **2.0 kg** sphere B which is at rest, and the coefficient of restitution for this collision is **0.50**. Sphere B is attached to the top end of a rod that pivots freely about point $O$ as shown. Sphere B is also attached to an **initially unstretched** linear spring with a stiffness of **52 N/m**. The rope and the rod are massless, and the sizes of the two spheres are negligible. Assume $\mathbf{g = 10\ m/s^2}$ and ignore friction and air resistance.




**Parts a) and b) below can be completed independently of each other.**

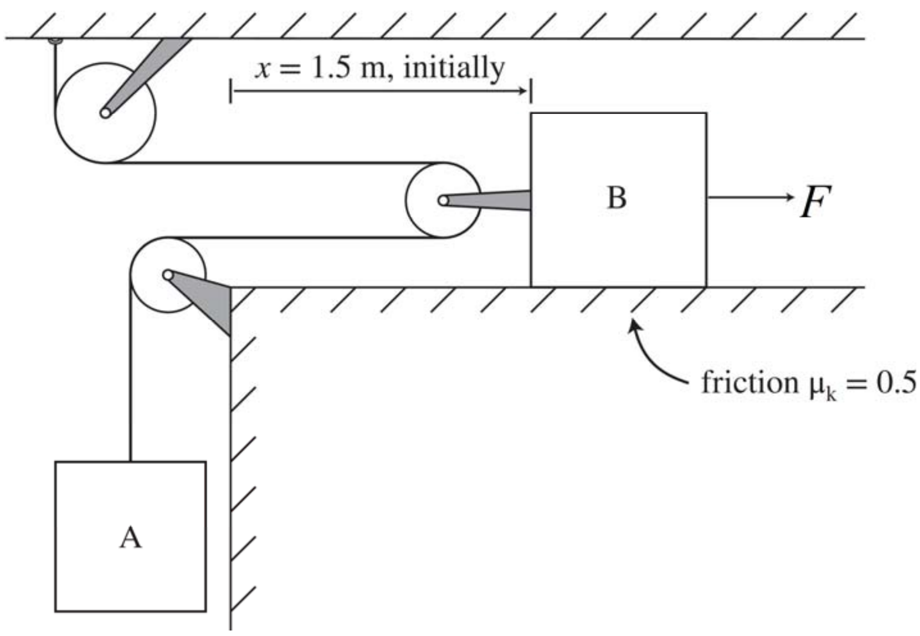
a) [7] If sphere B is moving at 4.8 m/s immediately after the collision, what was the release angle $\alpha$?

b) [8] At the instant $\theta = 30°$, what is the tension in the rod? The geometry at the instant $\theta = 30°$ is given by the diagram below with $L = 2.6$ m.



**Extra Page for Question 1**

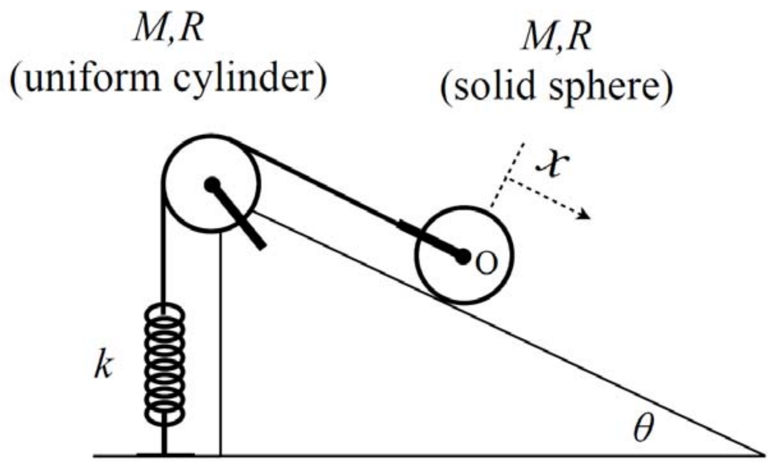
2. [7 marks] Block B (**$m_B$ = 8 kg**) sits on a rough surface, a distance $x$ = 1.5 m from the edge. It is attached on one side to block A (**$m_A$ = 4 kg**) by an unstretchable tether and a massless pulley system. On the other side, it is subjected to a rightward-directed force $F(x) = 2K(x_e - x) + 4Q(x_e - x)^3$, where **$K$ = 6.0 N/m**, **$Q$ = 0.4 N/m$^3$** and **$x_e$ = 2.5 m**. The coefficient of kinetic friction between block B and the rough surface is **$\mu_k$ = 0.5**. Block B is held stationary and then released. How fast is block A moving after it has fallen through a height of **2.0 m**? Use **g = 10 m/s$^2$** in your calculations.




You may use this integral: $$\int (a-x)^n \, dx = -\frac{(a-x)^{n+1}}{n+1} + C, \quad n \neq -1$$

**Extra Page for Question 2**

3. [14 marks] Consider the following system which is released from rest when the linear spring is unstretched. The rope is attached to the uniform solid sphere via a light yoke and frictionless axle that passes through O, so the sphere can rotate freely about O. The rope does not slip relative to the pulley. The cylinder and the sphere have the **same mass *M* and radius *R***. For some distance, the sphere rolls down the incline without slipping.




**Parts a) and b) below can be completed independently of each other.** For both parts, your answers depend on at most $M$, $g$, $\theta$, $k$, $R$ and $x$.

a) [6] **Use energy methods** to determine the speed of the sphere's center of mass as a function of $x$.

b) [8] Determine the magnitude of the static frictional force acting on the sphere as a function of $x$.

**Extra Page for Question 3**