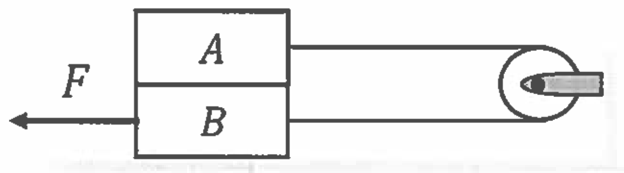
Q1

1. **[4 points]** Block A of mass m_A and block B of mass m_B are connected to each other through a pulley system as shown below. The masses of the pulley and the connection cable are negligible. A horizontal force F is applied to block B. Consider the different scenarios described below.



1) [1 point] The surface between A and B as well as the surface underneath block B are both smooth. Express the magnitude of the acceleration of block B in terms of one or more of the following parameters: F, m_A, m_B and g (gravitational acceleration). *No partial marks.*

$a_B = \dfrac{F}{m_A + m_B}$

2) The surface between A and B is rough, and the coefficients of static and kinetic friction are μ_s and μ_k, respectively. The surface underneath block B is smooth. Answer questions 2a) and 2b) below.

2a) [1 point] If the two blocks are observed to be stationary but about to move, express the force F in terms of one or more of the following parameters: m_A, m_B, g, μ_s and μ_k. *No partial marks.*

$F = 2\mu_s m_A g$

2b) [1 point] If the two blocks are observed to be in motion, express the magnitude of the acceleration of block B in terms of one or more of the following parameters: F, m_A, m_B, g, μ_s and μ_k. *No partial marks.*

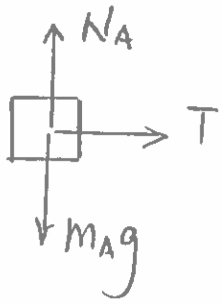
$a_B = \dfrac{F - 2\mu_k m_A g}{m_A + m_B}$

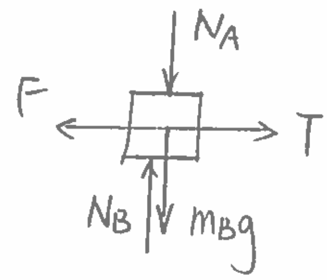
3) [1 point] The surface between A and B as well as the surface underneath block B are both rough. If the same force F as in 2b) is applied, will the magnitude of a_B be greater than, less than, or the same as the answer to 2b)? *No partial marks.*

a_B will be: (circle one)

Greater   Smaller   The same

1)

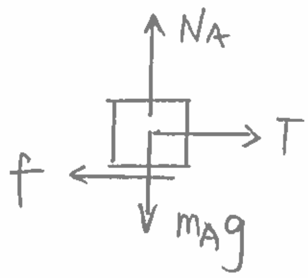


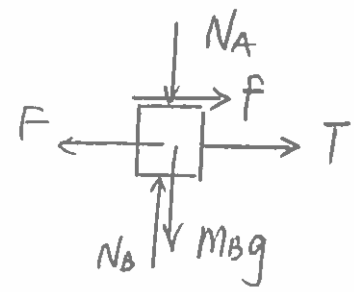


$$T = m_A a_B \qquad\qquad F - T = m_B a_B$$

$$\Rightarrow \quad a_B = \frac{F}{m_A + m_B}$$

2a)





$$T - \mu_s m_A g = 0 \qquad\qquad F - \mu_s m_A g - T = 0$$

$$\Rightarrow \quad F = 2\mu_s m_A g$$

2b)

$$T - \mu_k m_A g = m_A a_B \qquad\qquad F - \mu_k m_A g - T = m_B a_B$$

$$\Rightarrow \quad F - 2\mu_k m_A g = (m_A + m_B)\, a_B$$

$$\Rightarrow \quad a_B = \frac{F - 2\mu_k m_A g}{m_A + m_B}$$

Solution: Q2

1. At A, the total acceleration of the particle has a component to the right (a_t), and a component downwards (a_n). The total force is in the direction of the total acceleration, which is ↘ .

At B, a_n = 0 and a_t is downward. The total force is in the direction of the total acceleration, which is ↓ .

At C, the total acceleration of the particle has a component to the left (a_t), and a component upwards (a_n). The total force is in the direction of the total acceleration, which is ↖ .

At D, $a_n = 0$ and $a_t = 0$. The total force is 0.

2. At A, $mg = (1\text{ kg})(9.81\text{ m/s}^2) = 9.81\text{ N}$, $ma_n = (1\text{ kg})\frac{(6\text{ m/s})^2}{10\text{ m}} = 3.6\text{ N}$, so the normal force must be upwards ↑.

At B, no acceleration in the horizontal direction, normal force is 0.

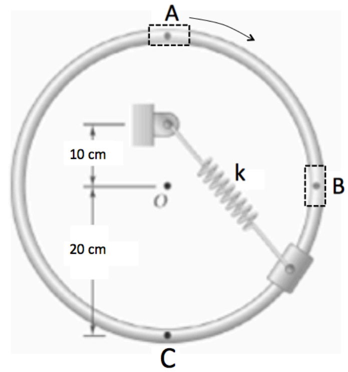
At C, normal force must be upwards ↑ to balance the gravity and provide the upward a_n.

At D, normal force is upwards ↑ to balance the gravity.

3. At A, $N_A = mg - mv_A^2 / r_A$. At B, $N_B = 0$. At C, $N_C = mg + mv_C^2 / r_C$. At D, $N_D = mg$. So the ranking is: C > D > A > B.

3. [5 points] No partial credit.

A collar starts at rest at location A and slides down along a smooth circular rod (radius r = 20.0 cm). The circular rod is oriented in a vertical plane, so that location A is at the top. The collar is attached to a spring; the other end of the spring is fixed 10.0 cm above the centre (O) of the circle as shown in the diagram. The resting length of the spring is 13.0 cm.



The collar slides down from A, reaches its maximum speed as it passes through location B, then continues to slide down through location C and beyond.

Consider the motion from location A through location C. In the questions below, circle the location(s) where the following conditions occur; circle "none" if there is no such location.

Clarification: Answer A means "at point A", whereas answer (A – B) means "at some location between A and B, excluding the endpoints".

Circle the location(s) where…

(a) [1] Elastic potential energy has its minimum value

A (A – B) B (B – C) C none

(b) [1] Total potential energy has its minimum value

A (A – B) B (B – C) C none

(c) [1] Total potential energy has its maximum value

A (A – B) B (B – C) C none

(d) [1] The horizontal component of acceleration is zero

A (A – B) B (B – C) C none

(e) [1] The vertical component of acceleration is zero

A (A – B) B (B – C) C none

ANSWERS:
(a) (A – B). The spring will be at its resting length somewhere between A and B.
(b) B. Energy is conserved, so total potential energy is minimum where speed is maximum.
(c) A. Kinetic energy is zero at A, so total potential energy is maximum.
(d) C. After passing location B the speed decreases as the spring stretches; speed will reach a (local) minimum at C, so tangential acceleration (horizontal in this case) is zero. Also, there is no horizontal component of force acting on the collar at this location.
(e) A and B. At A, v = 0 so $a_n = 0$. At B, speed is maximum, therefore tangential acceleration (vertical in this case) is zero.

Q4.

(i) [2 points] Conservation of linear momentum may occur in properly selected systems in specific directions. For the two systems, i.e. (a) Block and Ball and (b) Ball only, indicate in the table below whether linear momentum is conserved in the specified directions by circling "yes" or "no" in each cell. *One mark for each system. You must correctly answer for all four directions to get one mark. No partial marks.*

Is linear momentum conserved in the specified system and direction?	Direction (n)	Direction (t)	Horizontal direction	Vertical direction
(a) Block and Ball	yes no	yes no	yes no	yes no
(b) Ball	yes no	yes no	yes no	yes no

(ii) [2 points] If the impact is perfectly elastic, i.e. there is no loss of kinetic energy, and m_A is greater than m_B, mark the correct answers in the following table using "X".
One mark for each relation.

Relations between speeds	Yes	No	Uncertain
$\lvert V'_A\rvert < \lvert V_A\rvert$	X		
$\lvert V'_A\rvert^2 + \lvert V'_B\rvert^2 < \lvert V_A\rvert^2 + \lvert V_B\rvert^2$		X	

NOTE

(i) Linear momentum is conserved only in the marked directions.

(ii) For A, in the horizontal direction, $m_A V_A$ – (horizontal impulse from the ball) = $m_A V'_A$.
When V'_A is in the same direction with V_A (both positive), $|V'_A|$ is small then $|V_A|$.

Conservation of kinetic energy means
$m_A|V'_A|^2+m_B|V'_B|^2 = m_A|V_A|^2+m_B|V_B|^2$ or
$|V'_A|^2+|V'_B|^2 + (m_A/m_B - 1)|V'_A|^2 = |V_A|^2+|V_B|^2 + (m_A/m_B - 1)|V_A|^2$, or
$|V'_A|^2 + |V'_B|^2 = |V_A|^2+|V_B|^2 + (m_A/m_B - 1)(|V_A|^2-|V'_A|^2)$.

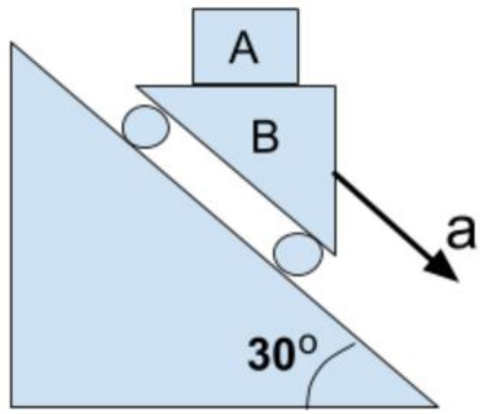
If $m_A > m_B$, with $|V'_A| < |V_A|$,
$|V'_A|^2 + |V'_B|^2 > |V_A|^2+|V_B|^2$.

5. [9 points] Cart B moves down a ramp with acceleration a = 2 m/s^2 . The surface between box A and cart B is horizontal and frictionless. Mass of box A is 10 kg. Answer the following questions:

a. Consider the horizontal component of motion for box A. Does it move horizontally, and if so, in which direction? Explain your answer.

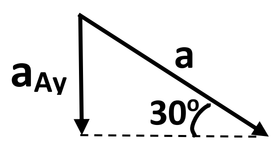
b. What is the acceleration of A (magnitude and direction)?

c. What is the magnitude of the normal force on A?



a) Since there is no friction force between A and B, box A cannot move horizontally on the surface of cart B, i.e. $\mathbf{a}_{Ax} = 0$ (where x is the horizontal direction). Box A can move only vertically down with acceleration $\mathbf{a}_{Ay}$ (where y is the vertical direction).

b) Forces acting on box A are the normal force N (up) and its weight W (down). The acceleration is $\mathbf{a}_{Ay}$ (down). $\mathbf{a}_{Ay}$ is the vertical component of the **acceleration a** of the cart B.

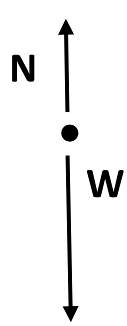


$\mathbf{a}_{Ay} = \mathbf{a} \sin 30^o = (2m/s^2) \sin 30^o = 1m/s^2$

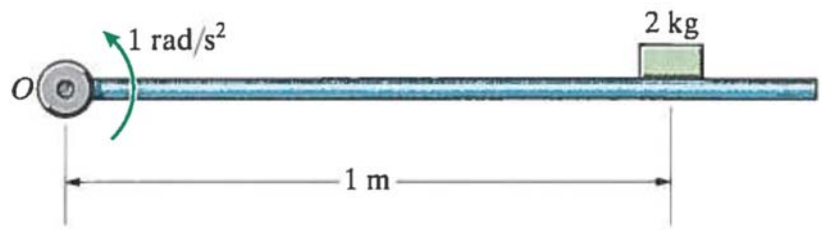
c) Equation of motion along y: +y points down and a_{Ay} points down;

$+W - N = +m_A a_{Ay}$ $\quad N = W - m_A a_{Ay} = m_A g - m_A a_{Ay} = m_A (g - a_{Ay})$

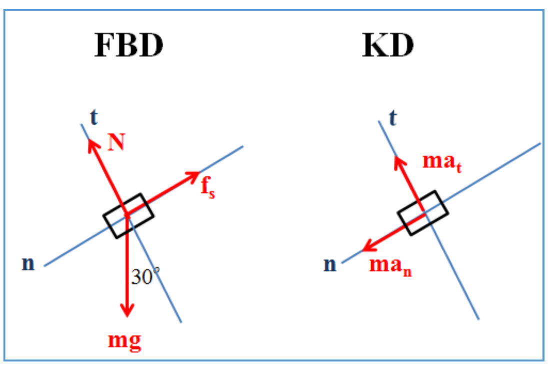
$N = (10kg)(9.81\ m/s^2 - 1\ m/s^2) = (10kg)(8.81\ m/s^2) = 88.1\ N$



6. [8 points] A 2-kg mass rests on a flat horizontal bar. The bar begins rotating in the vertical plane about O with a constant angular acceleration of 1 rad/s^2. The mass is observed to start slipping towards O when the bar is 30º above the horizontal. What is the coefficient of static friction between the mass and the bar?



Solution:



Rotation of the bar with constant angular acceleration:

$$2\alpha\theta = \omega^2 \quad \Rightarrow \quad 2(1)\left(\frac{\pi}{6}\right) = \omega^2 \quad \Rightarrow \quad \omega = 1.023 \text{ rad/s}$$

Normal acceleration of the mass: $a_n = \omega^2 r = 1.047 \text{ m/s}^2$

Tangential acceleration of the mass: $a_t = \alpha r = 1 \text{ m/s}^2$

Equations of motion:

$$+\nwarrow \quad N - mg\cos 30^o = ma_t \quad \Rightarrow \quad N = mg\cos 30^o + ma_t = 18.991\,\text{N}$$

$$+\swarrow \quad mg\sin 30^o - f_s = ma_n \quad \Rightarrow \quad f_s = mg\sin 30^o - ma_n = 7.716\,\text{N}$$

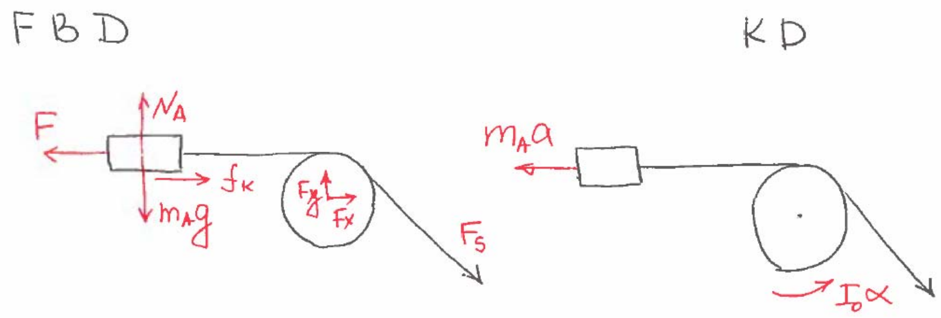
Because the mass starts to slip, $f_s = \mu_s N \quad \Rightarrow \quad \mu_s = \dfrac{f_s}{N} = 0.406$.

7. [10 points] A 2-kg block *A* is connected to the spring by an inextensible string of negligible mass passing over a pulley without slipping. The spring has a constant of $k = 3$ N/m and is initially unstretched. The pulley is a uniform solid cylinder with radius $R = 0.2$ m and mass $M = 4$ kg. A constant horizontal force $F = 50$ N is applied to block *A* such that the entire system starts to move from rest. The coefficient of kinetic friction between block *A* and the surface is $\mu_k = 0.1$.

Determine the speed of block *A* when it has moved a distance $S_A = 0.4$ m.

Solution.
Method 1: Work and energy principle for the entire system (block + pulley)
Method 2: Block and pulley separately: equations of motions
    2a: work and energy principle for block only
    2b: kinematics of the rectilinear motion of the block

Method 1.



Principle of work and energy

$$\Sigma U_{1-2} = \Sigma T_2 - \cancel{\Sigma T_1}$$

(motion from rest)

Kinetic energy: $T_A = \frac{1}{2} m_A v_A^2$

$$T_P = \frac{1}{2} I_o \omega^2$$

Inextensible cable: $v_A = \omega R \Rightarrow \omega = \frac{v_A}{R}$

Moment of inertia: $I_o = \frac{1}{2} M R^2$

$$\Sigma T_2 = T_A + T_P = \frac{1}{2} m_A v_A^2 + \frac{1}{2} I_o \omega^2$$

$$= \frac{1}{2} m_A v_A^2 + \frac{1}{2} \cdot \frac{1}{2} M R^2 \cdot \frac{v_A^2}{R^2} = \frac{1}{4} v_A^2 (2 m_A + M)$$

Forces that do work: F, f_K, F_S

$$U_F = F S_A \cos 0^\circ = F S_A$$

$$U_{f_K} = - f_K S_A = - \mu_K m_A g S_A$$

Force balance in y-direction: $N_A = m_A g \Rightarrow f_K = \mu_K m_A g$

$$U_{F_S} = - \frac{1}{2} k S_A^2$$

$$\Sigma U_{1-2} = F S_A - \mu_K m_A g S_A - \frac{1}{2} k S_A^2$$

$$= \Sigma T_2$$

$$F S_A - \mu_K m_A g S_A - \frac{1}{2} k S_A^2 = \frac{1}{4} v_A^2 (2 m_A + M)$$

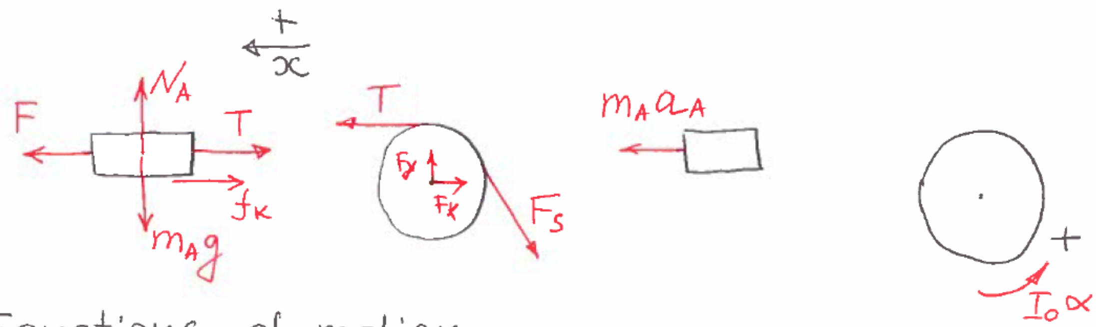
$$v_A = 2 \left[\frac{F S_A - \mu_K m_A g S_A - \frac{1}{2} k S_A^2}{2 m_A + M} \right]^{0.5}$$

$$v_A = 2 \left[\frac{50 \cdot 0.4 - 0.1 \cdot 2 \cdot 0.4 \cdot 9.81 - \frac{1}{2} 3 \cdot 0.4^2}{2 \cdot 2 + 4} \right]^{0.5}$$

$$v_A = 3.08 \ \frac{m}{s}$$

Method 2.

FBD KD



Equations of motion.

Block: $F - T - f_k = m_A a_A \quad (1)$ $\quad f_k = \mu_k N_A = \mu_k m_A g$

$N_A - m_A g = 0$ $\quad F_s = k s_A$ (not const)

Pulley: $TR - F_s R = I_o \alpha \quad (2)$ $\quad I_o = \frac{1}{2} M R^2 = 0.08 \text{ kg}\cdot\text{m}^2$ $\quad \alpha = \frac{a_A}{R}$

Unknowns: a_A, T

Eq (1): $F - T - \mu_k m_A g = m_A a_A \Rightarrow T = F - \mu_k m_A g - m_A a_A$

Eq (2): $TR - k s_A R = I_o \frac{a_A}{R}$ ← sub in

$$R(F - \mu_k m_A g - m_A a_A) - k s_A R = I_o \frac{a_A}{R}$$

$$a_A = \frac{RF - R\mu_k m_A g - k s_A R}{\frac{I_o}{R} + R m_A} = \frac{F - \mu_k m_A g - k s_A}{\frac{I_o}{R^2} + m_A}$$

$$T = F - \mu_k m_A g - m_A \cdot \frac{F - \mu_k m_A g - k s_A}{\frac{I_o}{R^2} + m_A}$$

$$T = \frac{(F - \mu_k m_A g)\left(\frac{I_o}{R^2} + m_A\right) - m_A (F - \mu_k m_A g) + m_A k s_A}{\frac{I_o}{R^2} + m_A}$$

$$T = \frac{\frac{I_o}{R^2}(F - \mu_k m_A g) + m_A k s_A}{\frac{I_o}{R^2} + m_A}$$

a_A and T are functions of s_A.

↓

not const acceleration

Method 2a:

Work and energy principle for block only:

$$\sum U_{1-2} = \sum T_2 - \underbrace{\cancel{\sum T_1}}_{0}$$

Kinetic energy: $T_2 = \frac{1}{2} m_A v_A^2$

Forces that do work: F, T, f_k

$$U_{1-2} = \int_0^{s_A} (F - T - f_k)\, ds =$$

$$= \int_0^{s_A} \left(F - \frac{\frac{I_o}{R^2}(F - \mu_k m_A g) + m_A k\, s_A}{\frac{I_o}{R^2} + m_A} - \mu_k m_A g \right) ds$$

$$= F s_A - \frac{\frac{I_o}{R^2}(F - \mu_k m_A g)}{\frac{I_o}{R^2} + m_A} s_A - \frac{m_A k}{\frac{I_o}{R^2} + m_A} \frac{s_A^2}{2}$$

$$- \mu_k m_A g s_A = \frac{1}{2} m_A v_A^2$$

$$v_A = \left\{ \frac{2}{m_A} \left[F s_A - \frac{\frac{I_o}{R^2}(F - \mu_k m_A g)}{\frac{I_o}{R^2} + m_A} s_A - \frac{m_A k}{\frac{I_o}{R^2} + m_A} \frac{s_A^2}{2} - \mu_k m_A g s_A \right] \right\}^{0.5}$$

$$v_A = \left\{ \frac{2}{2} \left[50 \cdot 0.4 - \frac{\frac{0.08}{0.2^2}(50 - 0.1 \cdot 2 \cdot 9.81)}{\frac{0.08}{0.2^2} + 2} 0.4 - \frac{2 \cdot 3}{\frac{0.08}{0.2^2} + 2} \cdot \frac{0.4^2}{2} - 0.1 \cdot 2 \cdot 9.81 \cdot 0.4 \right] \right\}^{0.5}$$

$$v_A = 3.08 \ \frac{m}{s}$$

Method 2b:

Kinematics: rectilinear motion of block:

$$a\,ds = v\,dv$$

$$\int_0^{s_A} \frac{F - \mu_k m_A g - k s_A}{\frac{I_o}{R^2} + m_A}\, ds = \int_0^{v_A} v\,dv$$

$$\frac{1}{2} v_A^2 = \frac{F - \mu_k m_A g}{\frac{I_o}{R^2} + m_A} s_A - \frac{1}{2} \frac{k s_A^2}{\frac{I_o}{R^2} + m_A}$$

$$v_A = \left[\frac{2(F - \mu_k m_A g)}{\frac{I_o}{R^2} + m_A} s_A - \frac{k s_A^2}{\frac{I_o}{R^2} + m_A} \right]^{0.5}$$

$$v_A = \left[\frac{2(50 - 0.1 \cdot 2 \cdot 9.81)}{\frac{0.08}{0.2^2} + 2} 0.4 - \frac{3 \cdot 0.4^2}{\frac{0.08}{0.2^2} + 2} \right]^{0.5}$$

$$v_A = 3.08 \ \frac{m}{s}$$

Solution Q8

We begin by determining the speed u with which the lighter ball strikes the heavier one. Energy conservation yields

$$\tfrac{1}{2} M_2 v^2 = \tfrac{1}{2} M_2 u^2 + M_2 g h, \text{ where } h = 2.8\ m,$$

whence

$$u^2 = v^2 - 2gh$$

Now we have to look at the impact between the two balls. Let u' and v' be the velocities (taken as positive in the upward direction) of the lighter and heavier ball, respectively, after the collision. Momentum conservation yields

$$M_2\, u = M_2\, u' + M_1\, v'$$

while the information about the coefficient of restitution implies

$$v' - u' = e\, u\,, \qquad \text{with } e = 0.57.$$

On rewriting the first equation as

$$u = u' + (M_1 / M_2)\, v'$$

and on adding the two equations we obtain

$$[1 + (M_1 / M_2)]\, v' = (1 + e)\, u$$

i.e.,

$$v' = [(1 + e)\, u]\ [1 + (M_1 / M_2)]^{-1}$$

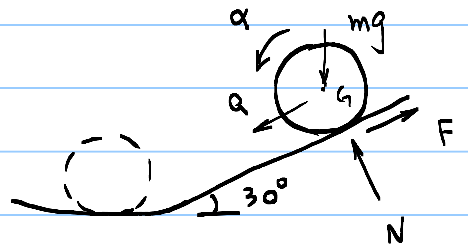
The maximum height is given by

$$h + v'^2/(2g) = h + (1 + e)^2 [(v^2 /2g) - h]\ [1 + (M_1 / M_2)]^{-2} =$$
$$2.8 + (1.57)^2\ [(196 /19.62) - 2.8][1 + (350/190)]^{-2} = 4.99\ m$$

Note Title

Q9

Solution



Moment of inertia

$$I_G = mk^2 = \frac{200}{32.2}\cdot\left(\frac{4}{12}\right)^2$$

$$= 0.69 \text{ slug}\cdot\text{ft}^2$$

(i)

$\swarrow$ $\quad mg\sin 30^\circ - F = ma$

$\circlearrowleft_G$ $\quad F\cdot\left(\frac{6}{12}\right) = I_G\,\alpha$

$\therefore \quad 200\times\frac{1}{2} - F = \frac{200}{32.2}\,a \qquad ①$

$F/2 = 0.69\,\alpha \qquad ②$

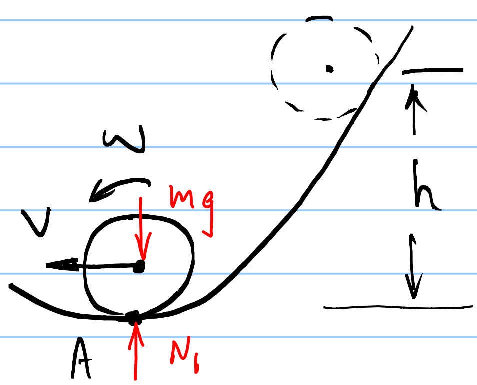
and $\quad a = \left(\frac{6}{12}\right)\cdot\alpha = \frac{1}{2}\alpha. \qquad ③$

$① \rightarrow \quad 100 - 1.38\alpha = \frac{100}{32.2}\alpha \Rightarrow \boxed{\alpha = 22.29 \text{ rad/s}^2 \circlearrowright}$

(ii) Conservation of energy

$$mgh = \frac{1}{2}mV^2 + \frac{1}{2}I_G\omega^2$$

with $h = 1.2 - \left(\frac{6}{12}\right) = 0.7$ ft, $\quad \omega = V/\left(\frac{6}{12}\right) = 2V$



$$200\times 0.7 = \frac{1}{2}\,\frac{200}{32.2}V^2 + \frac{1}{2}0.69\cdot 4\cdot V^2$$

$$V^2 = 31.21$$

$$\boxed{V = 5.59 \text{ ft/s} \leftarrow}$$

(iii) At position A, $\quad N_1 - mg = m\cdot V^2/R$, $\quad R = 2 - \left(\frac{6}{12}\right) = 1.5$ ft,

$$N_1 = mg + mV^2/R = 200 + \frac{200}{32.2}\,31.21/1.5 = \boxed{329.23 \text{ lb} \uparrow}$$