Faculty of Engineering and Department of Physics

Engineering Physics 131

Final Examination

Saturday April 27, 2019; 9:00 am – 11:30 am

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1. Closed book exam. No textbook or notes allowed.
2. Formula sheets are included (may be removed).
3. The exam has 9 problems and is out of 65 points. Attempt all parts of all problems.
4. *Questions 1 to 4 do not require detailed calculations. Only the final answers will be marked.*
5. For Questions 5 to 9, details and procedures to solve these problems will be marked. Show all work in a clear and logical manner.
6. Write your solution directly on the pages with the questions. Indicate clearly if you use the backs of pages for material to be marked.
7. Give answers with **three significant figures**, and **box your final answer**.
8. Only non-programmable calculators approved by the Faculty of Engineering are permitted. All other electronic devices (e.g. cell phones) must be turned off and stowed away out of sight.

DO NOT separate the pages of the exam containing the problems.

LAST NAME: ______________________________________

FIRST NAME: ______________________________________

ID#: ____

Please circle the name of your section's instructor:

EB01: Wheelock **EB02: Jung** **EB03: Komrakova**

EB04: Wang **EB05: Boninsegni** **EB06: Tang**

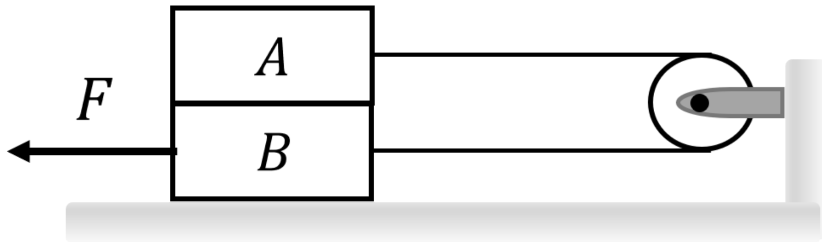
Address all inquiries to an invigilator. Do not communicate with other candidates. If you become ill during the exam, contact an invigilator immediately. (You may not claim extenuating circumstances and request your paper to be cancelled after writing and handing in your examination.) You may not leave the exam until at least 30 minutes have elapsed.

End of Exam: When the signal is given to end the exam, students must promptly cease writing. If a student does not stop at the signal, the instructor has the discretion either not to grade the exam paper or to lower the grade on the examination. **If you do not hand in your exam prior to 11:20 am, please remain seated until we finish collecting ALL the exams from the entire class.**

Please do not write in the table below

Question	**Value (points)**	**Mark**
1	4	
2	5	
3	5	
4	4	
5	9	
6	8	
7	10	
8	10	
9	10	
Total	**65**	

1. [4 points] Block A of mass m_A and block B of mass m_B are connected to each other through a pulley system as shown below. The masses of the pulley and the connection cable are negligible. A horizontal force F is applied to block B. Consider the different scenarios described below.



1) [1 point] The surface between A and B as well as the surface underneath block B are both smooth. Express the magnitude of the acceleration of block B in terms of one or more of the following parameters: F, m_A, m_B and g (gravitational acceleration). *No partial marks.*

$a_B =$ ____

2) The surface between A and B is rough, and the coefficients of static and kinetic friction are μ_s and μ_k, respectively. The surface underneath block B is smooth. Answer questions 2a) and 2b) below.

2a) [1 point] If the two blocks are observed to be stationary but about to move, express the force F in terms of one or more of the following parameters: m_A, m_B, g, μ_s and μ_k. *No partial marks.*

$F =$ ____

2b) [1 point] If the two blocks are observed to be in motion, express the magnitude of the acceleration of block B in terms of one or more of the following parameters: F, m_A, m_B, g, μ_s and μ_k. *No partial marks.*

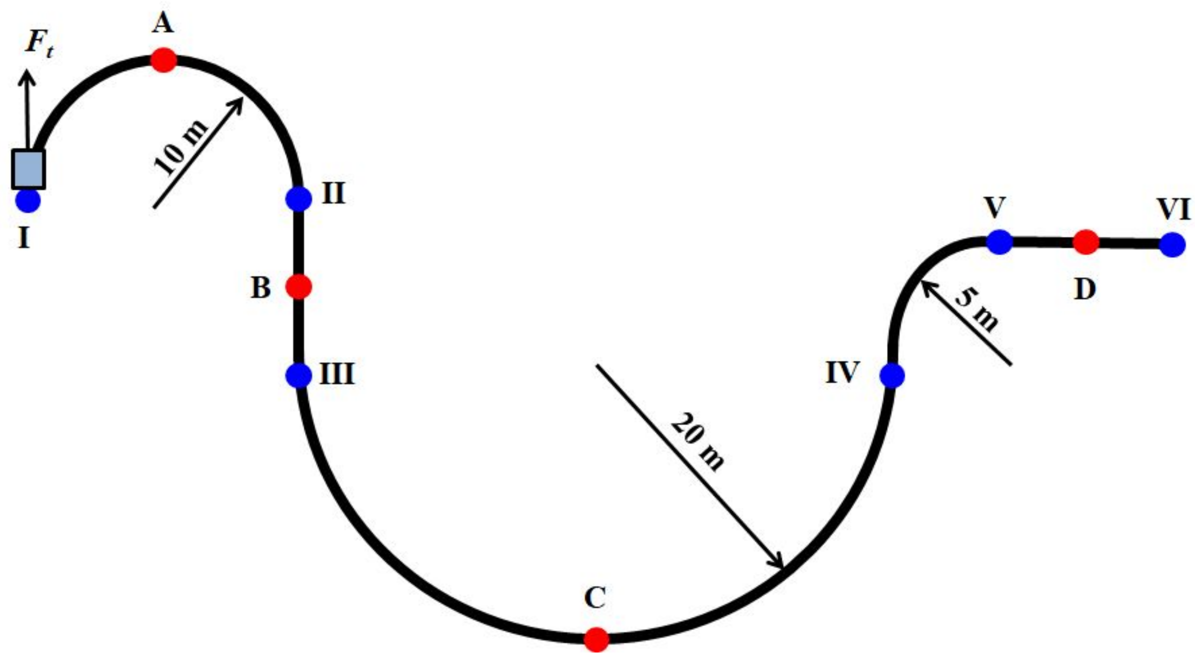
$a_B =$ ____

3) [1 point] The surface between A and B as well as the surface underneath block B are both rough. If the same force F as in 2b) is applied, will the magnitude of a_B be greater than, less than, or the same as the answer to 2b)? *No partial marks.*

a_B will be: (circle one)

Greater Smaller The same

2. [5 points] A 1-kg collar travels along the smooth curved bar in the vertical plane, as shown below. The bar is a semicircle of radius 10 m from I to II, a semicircle of radius 20 m from III to IV, and a quarter-circle of radius 5 m from IV to V. From II to III and from V to VI, the bar is straight. Points **A**, **B**, **C** and **D** are respectively the middle points of segments (I, II), (II, III), (III, IV) and (V, VI). A force F_t , which varies in time, is applied in the tangential direction so that the collar's speed increases between I and III, decreases between III and V, and is constant between V and VI. The speed of the collar is 6 m/s at **A**, 8 m/s at **B**, 4 m/s at **C**, and 2 m/s at **D**.



(a) Circle the approximate direction of **the total force** on the collar for each of the points below. If the total force is zero, circle 0. [0.5 points for each correct answer, no partial marks.]

At **A**: → ← ↑ ↓ ↗ ↘ ↖ ↙ 0

At **B**: → ← ↑ ↓ ↗ ↘ ↖ ↙ 0

At **C**: → ← ↑ ↓ ↗ ↘ ↖ ↙ 0

At **D**: → ← ↑ ↓ ↗ ↘ ↖ ↙ 0

(b) Circle the approximate direction of **the normal force exerted by the bar** on the collar for each of the points below. If the normal force is zero, circle 0. [0.5 points for each correct answer, no partial marks.]

At **A**: → ← ↑ ↓ ↗ ↘ ↖ ↙ 0

At **B**: → ← ↑ ↓ ↗ ↘ ↖ ↙ 0

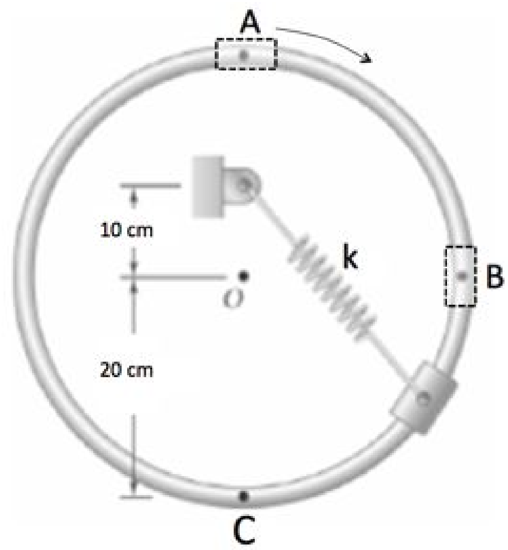
At **C**: → ← ↑ ↓ ↗ ↘ ↖ ↙ 0

At **D**: → ← ↑ ↓ ↗ ↘ ↖ ↙ 0

(c) Rank **the magnitude of the normal force from the bar** on the collar at the four points **A**, **B**, **C**, **D**, from the largest to the smallest and indicate any ties. [1 point for the correct answer, no partial marks.]

3. [5 points] No partial credit.

A collar starts at rest at location A and slides down along a smooth circular rod (radius r = 20.0 cm). The circular rod is in a vertical plane, so that A is at the top, B is the rightmost point, and C is at the bottom. The collar is attached to a spring; the other end of the spring is fixed 10.0 cm above the centre (O) of the circle as shown in the diagram. The undeformed length of the spring is 13.0 cm.



The collar slides down from A, reaches its maximum speed as it passes through B, then continues to slide down through C and beyond.

Consider the motion from A through C. In each question below, circle the location(s) where the stated condition occurs; circle "none" if there is no such location.

Clarification: Answer A means "at point A", whereas answer (A – B) means "at some location between A and B, excluding the endpoints".

Circle the location(s) where…

(a) [1] Elastic potential energy has its minimum value

A (A – B) B (B – C) C none

(b) [1] Total potential energy has its minimum value

A (A – B) B (B – C) C none

(c) [1] Total potential energy has its maximum value

A (A – B) B (B – C) C none

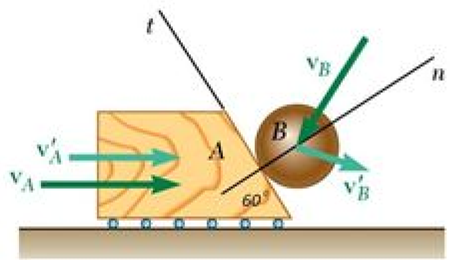
(d) [1] The horizontal component of acceleration is zero

A (A – B) B (B – C) C none

(e) [1] The vertical component of acceleration is zero

A (A – B) B (B – C) C none

4. [4 points] Consider the impact between block A of mass m_A and ball B of mass m_B. Block A moves freely on the horizontal surface, i.e., no friction. Before the impact the velocities of A and B are V_A and V_B, respectively. After the impact the velocities of A and B are V'_A and V'_B, respectively. The directions of the velocities are as shown. The contact surface is frictionless. Direction n is used to denote the direction normal (perpendicular) to the contact surface, and t is used to denote the direction tangential (parallel) to the contact surface.



(i) [2 points] Conservation of linear momentum may occur in properly selected systems in specific directions. For the two systems, i.e. (a) Block and Ball and (b) Ball only, indicate in the table below whether linear momentum is conserved in the specified directions by circling "yes" or "no" in each cell.
One mark for each system. You must correctly answer for all four directions to get one mark. No partial marks.

Is linear momentum conserved in the specified system and direction?	Direction (n)	Direction (t)	Horizontal direction	Vertical direction
(a) Block and Ball	yes no	yes no	yes no	yes no
(b) Ball	yes no	yes no	yes no	yes no

(ii) [2 points] If the impact is perfectly elastic, i.e. there is no loss of kinetic energy, and m_A is greater than m_B, mark the correct answers in the following table using "X".
One mark for each relation.

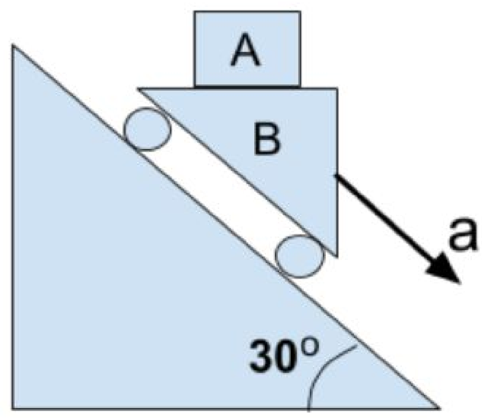
Relations between speeds	Yes	No	Uncertain
$\lvert V'_A\rvert < \lvert V_A\rvert$			
$\lvert V'_A\rvert^2 + \lvert V'_B\rvert^2 < \lvert V_A\rvert^2 + \lvert V_B\rvert^2$			

5. [9 points] Cart B moves down a ramp with acceleration a = 2 m/s^2 . The surface between box A and cart B is horizontal and frictionless. Mass of box A is 10 kg. Answer the following questions:

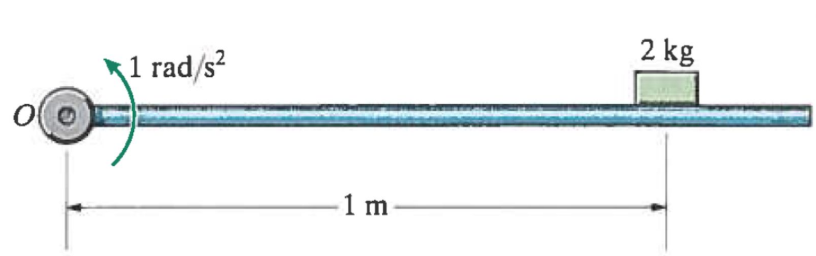
(a) Consider the horizontal component of motion for box A. Does it move horizontally, and if so, in which direction? Explain your answer.

(b) What is the acceleration of A (magnitude and direction)?

(c) What is the magnitude of the normal force on A?

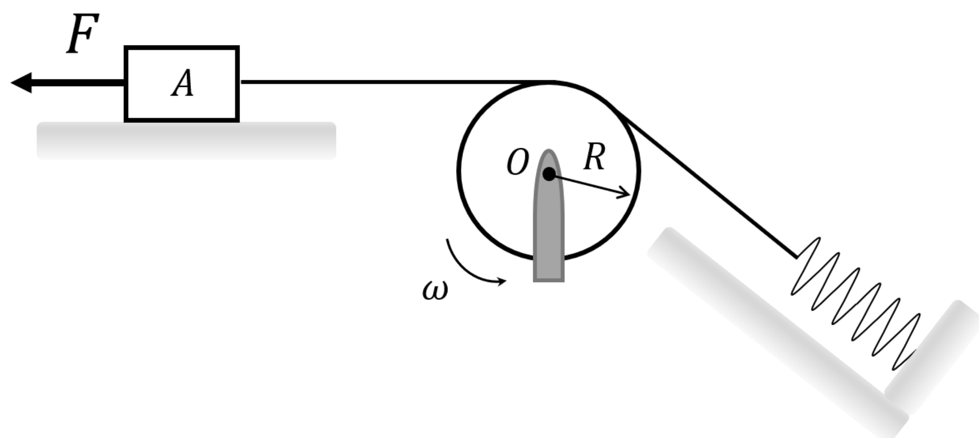


6. [8 points] A 2-kg mass rests on a flat horizontal bar. The bar begins rotating in the vertical plane about O with a constant angular acceleration of 1 rad/s^2. The mass is observed to start slipping towards O when the bar is 30º above the horizontal. What is the coefficient of static friction between the mass and the bar?

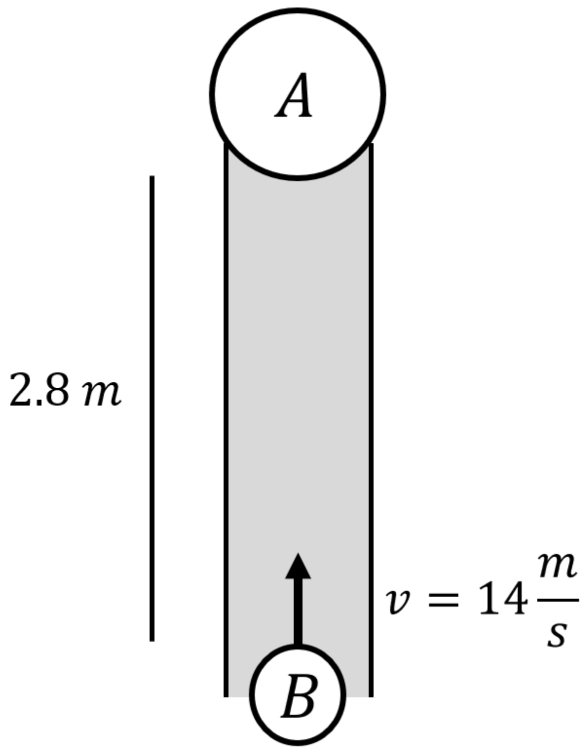


7. [10 points] A 2-kg block A is connected to the spring by an inextensible string of negligible mass passing over a pulley without slipping. The spring has a constant of k = 3 N/m and is initially unstretched. The pulley is a uniform solid cylinder with radius R = 0.2 m and mass M = 4 kg. A constant horizontal force F = 50 N is applied to block A such that the entire system starts to move from rest. The coefficient of kinetic friction between block A and the surface is μ_k= 0.1.

Determine the speed of block A when it has moved a distance S_A= 0.4 m.



8. [10 points] A ball A of mass $m_A = 0.35$ kg sits at rest on the upper end of a vertical, cylindrical pipe (see figure below). A second ball B of mass $m_B = 0.19$ kg, whose radius is smaller than that of the pipe, is fired up vertically from the bottom of the pipe with initial speed $v = 14$ m/s, and travels a distance of 2.8 m before striking ball A. Assuming a coefficient of restitution for the impact equal to 0.57, determine the maximum height to which ball A rises (with respect to the bottom of the pipe, i.e., initial position of ball B), as a result of being struck by ball B.



9. [10 points] A wheel weighs 200-lb, has a radius of $r = 6''$ (inches) and a radius of gyration of $k = 4''$ (inches) about its center. It is released from rest on a 30º incline at the position shown. The wheel rolls down the incline and the circular path (radius = 2 ft) without slipping. Determine

(i) [4 marks] the angular acceleration of the wheel when it is just released,

(ii) [4 marks] the velocity of the centre of the wheel when it rolls past position A,

(iii) [2 marks] the normal reaction force under the wheel when it rolls past position A.

*Hint: From the geometry, when it is just released the centre of the wheel is **1.2 ft** above point A.*
Moment of Inertia about the centre of mass $I = mk^2$. 1 ft = 12 inches, g = 32.2 ft/s^2.

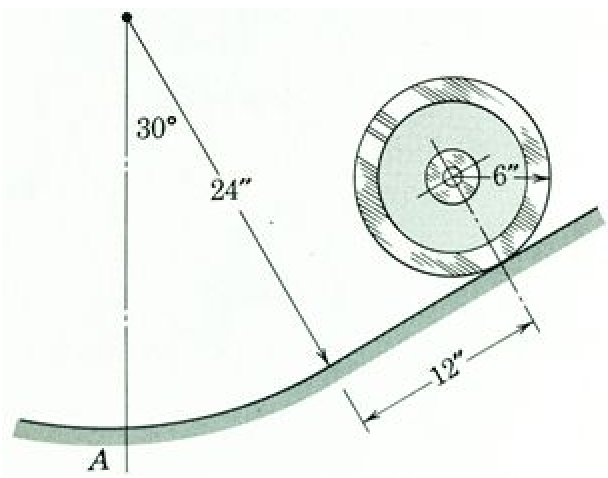
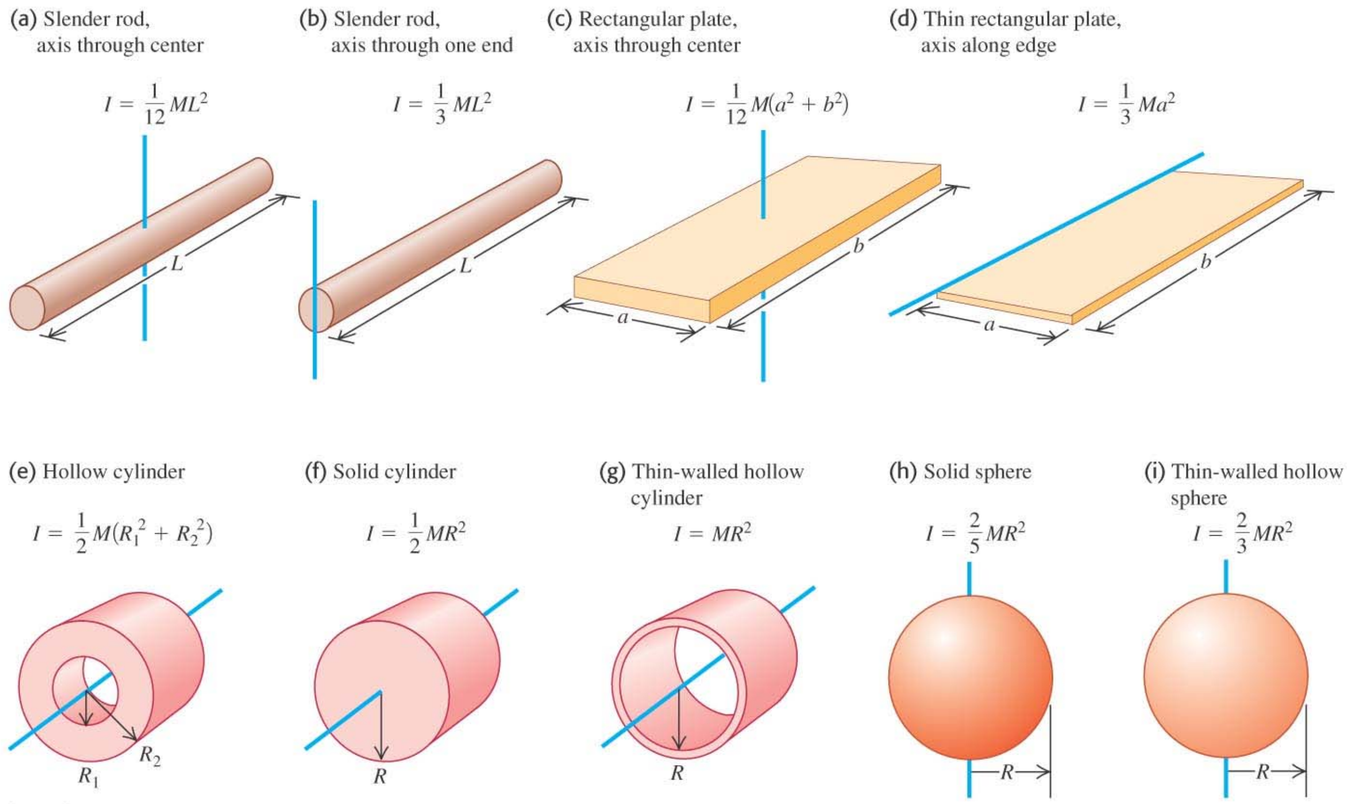


Table of Moment of Inertia



Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$	$a_t = \dot{v} = v\dfrac{dv}{ds}$
	$a_n = \dfrac{v^2}{\rho} \quad \rho = \dfrac{[1 + (dy/dx)^2]^{3/2}}{\lvert d^2y/dx^2 \rvert}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable $\mathbf{a}$	*Constant* $\mathbf{a} = \mathbf{a}_c$
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia $\quad I = \displaystyle\int r^2\,dm$

Parallel-Axis Theorem $\quad I = I_G + md^2$

Radius of Gyration $\quad k = \sqrt{\dfrac{I}{m}}$

Equations of Motion

Particle	$\Sigma\mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \displaystyle\int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \dfrac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \quad \epsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\displaystyle\int \mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\displaystyle\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\displaystyle\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\displaystyle\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\displaystyle\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$