Faculty of Engineering and Department of Physics

Engineering Physics 131

Final Examination

Tuesday April 12, 2022; 9:00 am – 11:30 am

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1. Closed book exam. No notes or textbooks allowed.
2. This is Part 2 of the exam, containing 5 questions each out of 10 marks, with a total of 50 marks. Attempt all questions.
3. The details and procedures to solve these problems will be marked. Show all work in a neat and logical manner. Give your answer in correct units with 3-digit accuracy.
4. Write your solution directly on the PDF file downloaded or write on papers and then convert to a **SINGLE PDF file.** Solutions to different questions must be written on different pages, i.e., DO NOT write solutions to different questions on the same page.
5. You must stop writing solutions at 11:30am. You will have until 11:40am to upload your solutions to **Common eClass**.

LAST NAME: ______________________________________

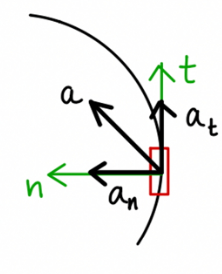
FIRST NAME: ______________________________________

ID#: ___________________

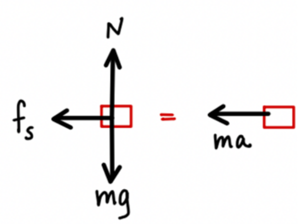
2-1. [10 marks]
A go-kart (total mass including driver = 150 kg) travels along a flat horizontal circular track with a radius of 25 m. Starting from rest, the cart increases its speed uniformly at a rate of 2.0 m/s^2. The cart continues to accelerate until it begins to skid off the track. The coefficient of static friction between the tires and the track is $\mu_s = 0.60$.
How many laps (or what fraction of a lap) around the track can it cover before it begins to skid? Your answer should be accurate to three significant figures (i.e., not asking for an integer).

$m = 150\,kg$
$r = 25\,m$
$a_t = 2.0\,ms^{-2}$
$\mu_s = 0.60$

View from above



Side view



$$F_{net} = f_s = ma$$

Cart begins to slide when $f_s = f_{s,max}$.

$$N = mg$$
$$f_{s,max} = \mu_s mg = ma$$
$$a = \mu_s g = (.6)(9.81) = 5.886\,ms^{-2}$$

Find the speed v_1 when the cart begins to slide:

$$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{a_t{}^2 + \left(\frac{v^2}{r}\right)^2}$$

$$v_1 = (\sqrt{r})(a^2 - a_t{}^2)^{\frac{1}{4}} = \sqrt{25}(5.886^2 - 2^2)^{.25} = 11.76\,ms^{-1}$$

Find the distance travelled along the track when v = v_1.

Method A: $$d = \frac{v^2}{2a_t} = \frac{11.76^2}{2(2)} = 34.6\,m$$

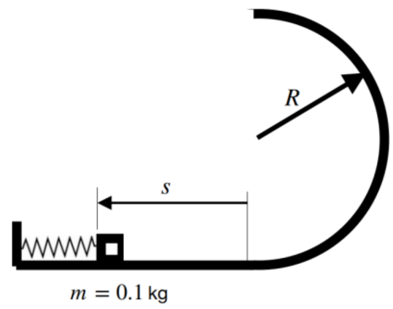
Method B: $$t = \frac{v}{a_t} = \frac{11.76}{2} = 5.882\,s$$
$$d = \frac{1}{2}a_t t^2 = (0.5)(2)(5.882^2) = 34.6\,m$$

Number of laps $= \frac{d}{2\pi r} = \frac{34.6}{2\pi(25)} = 0.220$ lap

2-2. [10 marks]

An m=0.1 kg mass is initially at rest at the end of a compressed spring of stiffness k = 20 N/m. After the spring is allowed to decompress, the mass slides over a frictionless vertically oriented, semi-circular track of radius R = 0.5 m.

Find the minimum compression, s, of the spring required so that the mass gets to the top of the track without leaving the track.



SOLN/ (ENERGY APPROACH)

ENERGY IS CONSERVED (NO NON-CONSERVATIVE FORCES)

INITIAL ENERGY: $\frac{1}{2}ks^2$

FINAL ENERGY: $\frac{1}{2}mv^2 + mgh$

WITH $h = 2R$

$$\Rightarrow \frac{1}{2}mv^2 + mg(2R) = \frac{1}{2}ks^2 \Rightarrow s^2 = \frac{m}{k}v^2 + 4\frac{m}{k}gR$$

IF JUST AT THE POINT OF LEAVING THE TRACK AT TOP: $g = \frac{v^2}{R}$

$$\Rightarrow s^2 = \frac{m}{k}(gR) + 4\frac{m}{k}gR = 5\frac{m}{k}gR$$

$$\Rightarrow s = \sqrt{5\frac{m}{k}gR}$$

$$\simeq \sqrt{5\frac{(0.1\,kg)}{(20\,N/m)}(9.81\,m/s^2)(0.5\,m)}$$

$$\Rightarrow s \simeq 0.350\,m$$

[FORCE APPROACH:

FOR SPEED UPON RELEASE BY SPRING: $F = -kx = ma \Rightarrow a = -\frac{k}{m}x$

$$v\,dv = a\,dx \Rightarrow \int_0^{V_0} v\,dv = \int_{-s}^{0} -\frac{k}{m}x\,dx \Rightarrow \frac{1}{2}V_0^2 = \frac{1}{2}\frac{k}{m}s^2 \Rightarrow V_0^2 = \frac{k}{m}s^2$$

FOLLOWING CIRCULAR TRACK

$$ma_t = -mg\sin\theta$$

$$\Rightarrow a_t = -g\sin\theta$$

$$v\,dv = a_t\,ds = -g\sin\theta\,ds = -g\sin\theta\,d(R\theta) = -gR\sin\theta\,d\theta$$

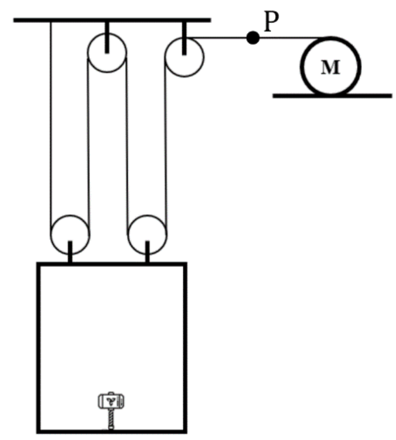
$$\Rightarrow \int_{V_0}^{V_f} V\,dV = \int_0^{\pi} -gR\sin\theta\,d\theta \Rightarrow \frac{1}{2}V_f^2 - \frac{1}{2}V_0^2 = +gR\cos\theta\Big|_0^{\pi} = -2gR$$

$$\Rightarrow V_f^2 = V_0^2 - 4gR = \frac{k}{m}s^2 - 4gR$$

(AS ABOVE) $V_f^2 = gR \Rightarrow s^2 = \frac{m}{k}(gR + 4gR) = 5\frac{m}{k}gR$ AS ABOVE $\Rightarrow s = 0.350\,m$]

2-3. [10 marks]

On a bet from his friends, Thor Odinson places his hammer in an elevator, as shown. The motor, *M*, lifts the elevator and hammer through the pulley system shown. The combined weight of the elevator and hammer is 2700 lbs. At the point P, indicated, the constant acceleration is 8 ft/s^2. The velocity of point P is 2 ft/s at time $t = 0$.



a) Starting at $t = 0$, what is the distance the elevator traveled in 2s? What is the velocity of the elevator at t = 2 s?

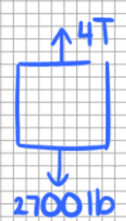
b) Starting at $t = 0$, what is the work done by the motor over the next 2s while the elevator rises?

Assume that the pulleys are ideal and massless.

(Solution-old version)

Solution

(a) FBD of elevator ⇒



$$\Sigma F_y = m a_y$$
$$4T - 2700 = \frac{2700}{32.2} a_e \quad \text{---- (1)}$$

From the pulley system ⇒

$$4 s_e + 1 s_m = l_T \Rightarrow 4 v_e + v_m = 0 \Rightarrow 4 a_e + a_m = 0$$

$$v_e = \frac{-v_m}{4} \quad \text{and} \quad a_e = \frac{-a_m}{4}$$

Since point P is along s_m, $v_p = v_m$ & $a_p = a_m$

$\therefore$ @ $v_m = 2$ ft/s and $a_m = 8$ ft/s^2, $v_e = -0.5$ ft/s & $a_e = -2$ ft/s^2

Using this value of a_e, T = 716.93 N

Over 2 seconds, the distance covered by the elevator is:

$$s_e = s_{eo} + v_{eo} t + \frac{1}{2} a_e t^2$$
$$= 0.5 * 2 + \frac{1}{2} * 2 * 2^2 = \underline{5 \text{ ft}}$$

Work done by motor = T * s_m

$$s_m = v_{mo} t + \frac{1}{2} a_m t^2$$
$$= 2 * 2 + \frac{1}{2} * 8 * 2^2$$
$$= \underline{20} \text{ ft (which is also } 4 s_e)$$

same as work done by elevator tension.

Work done by motor = T*20 = $\boxed{14300 \text{ lb·ft}}$

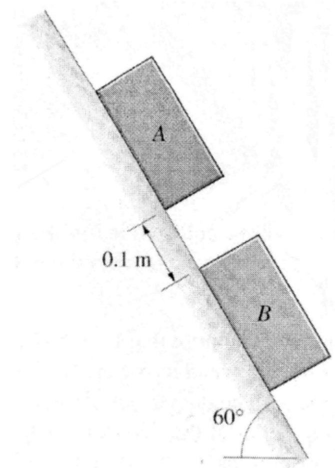
(b) $P_o = T \cdot v_m = 716.93 * 2 = 1433.86$ lb·ft/s = 2.61 hp

$$\varepsilon = \frac{P_o}{P_i} = \frac{2.61}{3.5} = \boxed{0.745}$$

2-4. [10 marks]

Crates A and B are both 5 kg, and the kinetic coefficients of friction between them and the inclined surface are $\mu_{kA} = 0.1$ and $\mu_{kB} = 0.4$, respectively. The coefficient of restitution between the crates upon collision is $e = 0.8$. The slope is at an angle of 60° from the horizontal. The crates are initially at rest and separated by 0.1 m, as shown.

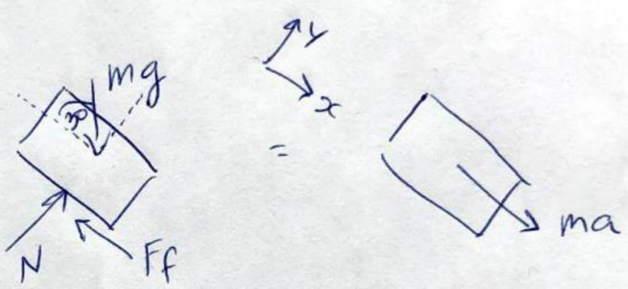
What are the speeds of crate A and crate B immediately after they collide?



$m_A = 5\,kg$ $\quad \mu_{sA} = 0.1$ $\quad e = 0.8$

$m_B = 5\,kg$ $\quad \mu_{sB} = 0.4$

find velocities before collision:



for A: $\Sigma F_x = m_A g \cos 30 - \mu_{kA} N = m_A a_A \quad (1)$

$\Sigma F_y = N_A - m_A g \cos 60 = 0 \quad (2)$

(2) into (1): $\cancel{m_A} g \cos 30 - \mu_{kA} \cancel{m_A} g \cos 60 = \cancel{m_A} a_A$

$a_A = 8.01\,m/s^2$

similarly for B: $N_B = m_B g \cos 60$

$\cancel{m_B} g \cos 30 - \mu_{kB} \cancel{m_B} g \cos 60 = \cancel{m_B} a_B$

$a_B = 6.53\,m/s^2$

when they collide $\Delta S_A = \Delta S_B + 0.1\,m$

$\cancel{v_{0A}}^{0} t + \frac{1}{2} a_A t^2 = \cancel{v_{0B}}^{0} t + \frac{1}{2} a_B t^2 + 0.1$

$\frac{1}{2}(8.01)t^2 = 0.1 + \frac{1}{2}(6.53)t^2$

$t = 0.3676\,s$

$\therefore v_A = \cancel{v_{0A}}^{0} + a_A t$ $\qquad \therefore v_B = \cancel{v_{0B}}^{0} + a_B t$

$\underline{v_A = 2.94\,m/s}$ $\qquad \underline{v_B = 2.40\,m/s}$

② solve for impact:

Conservation of Momentum for A+B:

$$\cancel{m_A} v_{A_1} + \cancel{m_B} v_{B_1} = \cancel{m_A} v_{A_2} + \cancel{m_B} v_{B_2}$$

$$2.94 + 2.40 = v_{A_2} + v_{B_2} \quad (3)$$

impact ·/· A + B:

$$e = \frac{v_{A_2} - v_{B_2}}{v_{B_1} - v_{A_1}}$$

$$0.8 = \frac{v_{A_2} - v_{B_2}}{2.40 - 2.94} \quad (4)$$

Solve (3) for v_{B_2} + plug into (4):

$$0.8 = \frac{v_{A_2} - (5.34 - v_{A_2})}{-0.54}$$

$$v_{A_2} = 2.45 \text{ m/s}$$

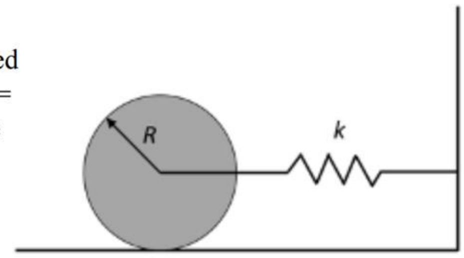
$$\vec{v}_{A_2} = 2.45 \text{ m/s} \; 60°$$

$$v_{B_2} = 2.89 \text{ m/s}$$

$$\vec{v}_{B_2} = 2.89 \text{ m/s} \; 60°$$

2-5. [10 marks]

A uniform disk of mass m = 100 kg and radius R = 0.75 m is attached to a fixed surface by a horizontal spring with a spring constant of k = 800 N/m. The disk is displaced to the right on the horizontal surface until the spring is compressed 0.5 m and then released from rest.

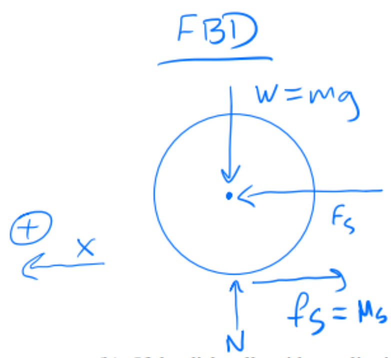


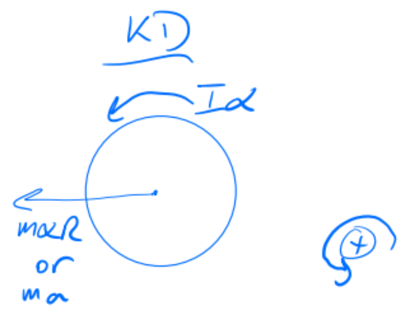
a) Draw the free body diagram and kinetic diagram of the disk.

b) If the disk rolls without slipping, what is its angular acceleration at the instant it is released?

c) What is the minimum coefficient of static friction for which the disk will not slip when it is released?

(a) Draw the free body diagram and kinetic diagram of the disk.





(b) If the disk rolls without slipping, what is its angular acceleration at the instant it is released?

$$\left.\begin{aligned} F_s - f_s &= m\alpha R \\ f_s R &= I\alpha \end{aligned}\right\}$$

$S_0 = 0.5\,m$

$$F_s - \frac{I\alpha}{R} = m\alpha R \Rightarrow F_s = m\alpha R + \frac{I\alpha}{R}$$

$$F_s = \alpha(mR + I/R)$$

$$\therefore \boxed{\alpha = \frac{F_s}{mR + I/R} = \frac{F_s R}{mR^2 + I}}$$

Now,

$$\alpha = \frac{F_s R}{mR^2 + I} = \frac{F_s R}{mR^2 + 1/2\, mR^2} = \frac{2F_s}{3mR} = \frac{2KS}{3mR}$$

via Moment of Inertia Table

$$\alpha = \frac{2(800)(0.5)}{3(100)(0.75)}$$

$$\alpha = 3.56\ \text{rad}/s^2$$

(c) What is the minimum coefficient of static friction for which the disk will not slip when it is released?

$$\left.\begin{aligned} F_s - f_s &= m\alpha R \\ f_s R &= I\alpha \end{aligned}\right\}$$

$S_0 = 0.5\,m$

$$f_s = \frac{I\alpha}{R} = \frac{I F_s}{mR^2 + I} = \frac{(1/2\, mR^2)KS}{mR^2 + (1/2\, mR^2)} = \frac{KS}{3}$$

$$\therefore f_s = \frac{(800)(0.5)}{3} = 133.3\,N$$

Now,

$$f_s = 133.3 \leq \mu_s mg = \mu_s (100)(9.81)$$

$$\therefore \mu_s = 0.136$$