Faculty of Engineering and Department of Physics

Engineering Physics 131

Midterm Examination

Saturday February 12, 2022; 14:00 – 15:30

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1. Closed book exam. No notes or textbooks allowed. Formula sheets are allowed.
2. This is Part 2 of the exam, containing 4 questions and is out of **36 points**. Attempt all questions.
3. *For Questions 2-1, 2-2, 2-3, 2-4, details and procedures to solve these problems will be marked.* Show all work in a neat and logical manner.
4. Write your solution directly on the PDF file downloaded or write on papers and then convert to a **SINGLE PDF file**, and upload to the exam page. Solutions to different questions must be written on different pages, i.e., DO NOT write solutions to different questions on the same page.
5. Write your Name and Student ID on the first page of your PDF file, and name the PDF file using your last name.

LAST NAME: ____

FIRST NAME: ____

ID#: ____________________

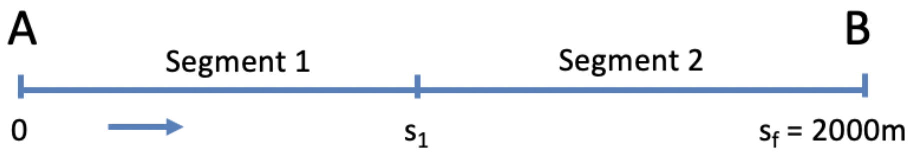
Please do not write in the table below.

Question	**Value (Points)**	**Mark**
2-1	9	
2-2	9	
2-3	9	
2-4	9	
Total	36	

2-1. [9 Points] A flying saucer wishes to fly from A to B, a distance of 2 km. It first accelerates at *0.8 g* (*g = 9.81 m/s²*) and then decelerates at *0.4 g*. Assume that it travels along a straight path directly from A to B, and that it starts and ends at *rest*.

(a) What maximum speed will it reach during the trip?
(b) Determine the travel time from A to B.

Solution. Erratic motion in two segments, each with constant acceleration. The saucer accelerates during segment 1, reaches a maximum speed of v_1 at location s_1, and then decelerates until it comes to rest at s = 2000 m. To maximize partial marks, variables for each segment must be clear and consistent throughout.



(a) Treat the acceleration and deceleration segments separately.

Let $a_1 = 0.8g$ and $a_2 = -0.4g$.
Let s_1 = position where deceleration begins, and $s_f = 2000\ m$.
Note that $v_0 = v_f = 0$, and v_1 = maximum speed, which occurs at s = s_1.

Segment 1: $v_1{}^2 = v_0{}^2 + 2a_1(s_1 - s_0) \rightarrow v_1{}^2 = 2a_1 s_1$

Segment 2: $v_f{}^2 = v_1{}^2 + 2a_2(s_f - s_1) \rightarrow v_1{}^2 = -2a_2(s_f - s_1)$

Set the two equations equal and solve for s_1.

$$2a_1 s_1 = -2a_2(s_f - s_1)$$
$$s_1(a_1 - a_2) = -a_2 s_f$$
$$s_1 = \frac{-a_2 s_f}{(a_1 - a_2)} = \frac{-(-.4)g(2000)}{(.8 - (-.4))g} = 667\ m$$
$$v_1 = \sqrt{2a_1 s_1} = \sqrt{2(.8)(9.81)(667)} = 102.3\ m/s$$

(b) Total travel time = time for segment one + time for segment two:

$$t_{tot} = t_1 + t_2$$
$$= \frac{v_1 - v_0}{a_1} + \frac{v_f - v_1}{a_2}$$
$$= \frac{102.3 - 0}{0.8(9.81)} + \frac{0 - 102.3}{-0.4(9.81)}$$
$$= 13.0 + 26.1 = 39.1\ s$$

2-2. [9 Points] Your EN PH professor, a big golf enthusiast, has promised you an A in the course if you can make what he claims is an impossible hole-in-one shot (a hole-in-one means hitting the golf ball into the flag hole in one shot). Unfortunately for you, today the wind is particularly crazy, creating an additional acceleration for the golf ball, *a*, in the direction shown in Figure 2-2.

Given that the launch speed of the ball is v_0, the direction of the initial velocity is $\vartheta = 30^0$, and the gravitational acceleration is $g = 9.81\ m/s^2$,

(i) if the coordinates (*x* and *y*) are defined as shown in Figure 2-2 and the ball is at the origin when t = 0 s, write down the horizontal and vertical coordinates *x* and *y* of the ball as functions of *t* and v_0, and

(ii) determine the initial velocity v_0 at which the ball must be shot to sink it.

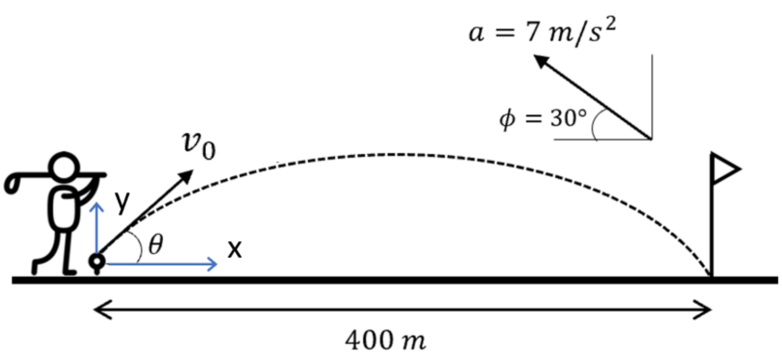


Figure 2-2

givens:-

$x_0 = y_0 = 0\,m$; $a_x = -7\cos 30$; $a_y = 7\sin 30 - g$

$y_f = 0$; $x_f = 400$; $v_{0x} = v_0\cos 30$; $v_{0y} = v_0 \sin 30$

(i) for horizontal motion:-

$x(v_0,t) = \cancel{x_0}^{0} + v_{0x}t + \frac{1}{2}a_x t^2$ ← sub values from givens

$x(v_0,t) = v_0 t\cos 30 - 3.5t^2\cos 30$

$$\boxed{x(v_0,t) = \frac{\sqrt{3}}{2}v_0 t - \frac{7\sqrt{3}}{4}t^2}$$

for vertical motion:-

$y(v_0,t) = \cancel{y_0}^{0} + v_{0y}t + \frac{1}{2}a_y t^2$ ← sub values from givens

$y(v_0,t) = \frac{1}{2}v_0 t + \frac{1}{2}\left(\frac{7}{2} - 9.81\right)t^2$

$$\boxed{y(v_0,t) = \frac{1}{2}v_0 t - 3.155t^2}$$

(ii) at t_f, $x_f = 400$, $y_f = 0$

$y(v_0,t_f) = 0 = \frac{1}{2}v_0\cancel{t_f} - 3.155t_f^{\cancel{2}} \Rightarrow -\frac{1}{2}v_0 = -3.155t_f \Rightarrow v_0 = 6.31t_f$

sub '$v_0 = 6.31t_f$' into $x(v_0,t_f)$ equation ⇒

$x(v_0,t_f) = 400 = \frac{\sqrt{3}}{2} * 6.31t_f^2 - \frac{7\sqrt{3}}{4}t_f^2$

$400 = 5.465t_f^2 - 3.031t_f^2$

$400 = 2.434t_f^2 \Rightarrow t_f = 12.8$ seconds

$v_0 = 6.31t_f = \boxed{80.9\,m/s}$

2-3. [9 Points] A particle moves on a curve path which is shown in Figure 2-3.

The particle's position vector as a function of time *t* is given by:

$$\vec{r}(t) = x(t)\vec{\imath} + y(t)\vec{\jmath} = [-0.31(t^2) + 7.2(t) + 28]\vec{\imath} + [0.22(t^2) - 9.1(t) + 30]\vec{\jmath}$$

with *r* in meters and time *t* in seconds and $\vec{\imath}$ and $\vec{\jmath}$ are the unit vectors in x and y directions.

(a) Find the velocity vector for the particle when t = 15 s in unit-vector notation. Then determine the normal and tangential components of the particle's velocity at that instant.

(b) Find the acceleration vector for the particle when t = 15 s as a magnitude and an angle measured counterclockwise from the positive x axis. Then determine the normal and tangential components of the particle's acceleration at that instant.

Hint: Find the angle of the acceleration vector measured counterclockwise from the positive n axis to calculate a_t and a_n.

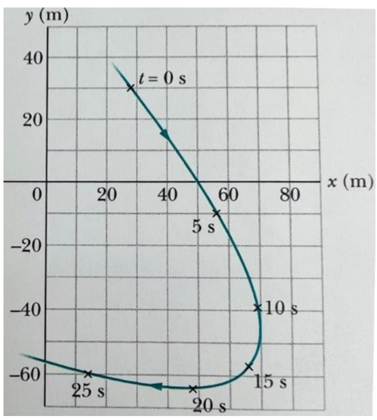


Figure 2-3

Solution

(a) Find the velocity vector for the particle in unit-vector notation. Determine the normal and tangential components of the particle's velocity when *t* = 15 s.

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28) = -0.62t + 7.2$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30) = 0.44t - 9.1$$

At *t* = 15 s, the equations above yield:

$$v_x = -2.1\ m/s$$

$$v_y = -2.5\ m/s$$

and therefore, the velocity vector is:

$$\vec{v} = -(2.1\ m/s)\hat{\imath} - (2.5\ m/s)\hat{\jmath}$$

To get the magnitude of $\vec{v}$:

$$v = [{v_x}^2 + {v_y}^2]^{1/2} = [(-2.1\ m/s)^2 + (-2.5\ m/s)^2]^{1/2} = 3.3\ m/s$$

Finally, since the velocity is always tangent to the path:

$$v_n = 0$$

$$v_t = 3.3\ m/s$$

(b) Find the acceleration vector for the particle as a magnitude and an angle. Determine the normal and tangential components of the particle's acceleration when *t* = 15 s. Hint: Find the angle of $\vec{a}$ measured counterclockwise from the positive *n* axis to calculate a_n and a_t.

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62\ m/s^2$$

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44\ m/s^2$$

and therefore, the acceleration vector is:

$$\vec{a} = -(0.62\ m/s^2)\hat{\imath} + (0.44\ m/s^2)\hat{\jmath}$$

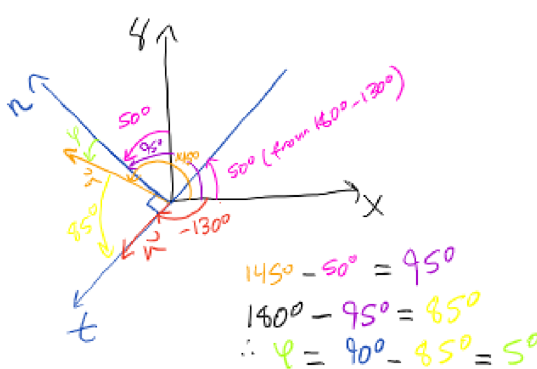
To get the magnitude and angle of $\vec{a}$:

$$a = [{a_x}^2 + {a_y}^2]^{1/2} = [(-0.62\ m/s^2)^2 + (0.44\ m/s^2)^2]^{1/2} = 0.76\ m/s^2$$

$$\beta = tan^{-1}\left(\frac{a_y}{a_x}\right) = tan^{-1}\left(\frac{0.44\ m/s^2}{-0.62\ m/s^2}\right) = 145^{\circ}$$

[$\beta = 145^{\circ}$ is not what is displayed on the calculator, but rather $\beta = -35^{\circ}$ is displayed on the calculator (which has the same tagent as $\beta = 145^{\circ}$). By inspection of the signs of the components of velocity components, we can rule out $\beta = -35^{\circ}$ as the desired angle of the acceleration which lies in the 2nd quadrant (and is given by $-35° + 180° = 145°$).]

From the figure below, $\varphi = 5°$.



Therefore, the normal and tangential components of the particle's acceleration are:

$$a_n = (5°) = 0.76\ m/s^2$$
$$a_t = (5°)\ = 0.066\ m/s^2$$

2-4. [9 Points] Car A is being pulled by truck B. If B is moving to the right with a speed of 6 m/s, and decreasing in speed at a rate of *0.9 m/s²*, determine the velocity and acceleration of car A, and the velocity and acceleration of A relative to B.

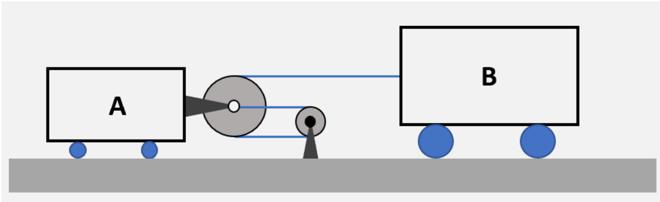
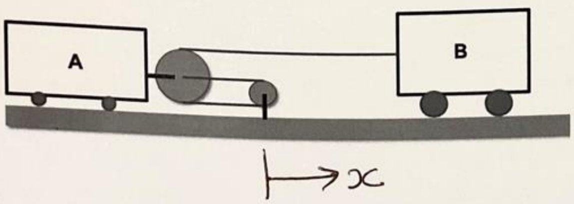


Figure 2-4

Solution



given

$v_B = 6\,m/s$

$a_B = -0.9\,m/s^2$

find

$v_A, a_A, v_{A/B}, a_{A/B}$

length of cable:

$l = x_B - 3x_A + C$

$dl/dt = v_B - 3v_A = 0 \quad (1)$

$d^2l/dt^2 = a_B - 3a_A = 0 \quad (2)$

v_B into eq'n 1: $6 - 3v_A = 0$

$\underline{\underline{v_A = 2\,m/s}}$ (ie. to the right)

a_B into eq'n 2: $-0.9 - 3a_A = 0$

$\underline{\underline{a_A = -0.3\,m/s^2}}$ (ie. slowing)

$v_{A/B} = v_A - v_B = 2 - 6$

$\underline{\underline{v_{A/B} = -4\,m/s}}$ (ie. to the left)

$a_{A/B} = a_A - a_B = -0.3 + 0.9$

$\underline{\underline{a_{A/B} = 0.6\,m/s^2}}$ (ie. to the right)