

Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	Constant $a = a_c$
$a = \frac{dv}{dt}$	$v = v_0 + a_c t$
$v = \frac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$a ds = v dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates	r, θ, z Coordinates
$v_x = \dot{x}$ $a_x = \ddot{x}$	$v_r = \dot{r}$ $a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$ $a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$ $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$ $a_z = \ddot{z}$	$v_z = \dot{z}$ $a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$	$a_t = \dot{v} = v \frac{dv}{ds}$
	$a_n = \frac{v^2}{\rho}$ $\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{ d^2y/dx^2 }$

Relative Motion

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Rigid Body Motion About a Fixed Axis

Variable a	Constant $a = a_c$
$\alpha = \frac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \frac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_c t^2$
$\omega d\omega = \alpha d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$$

Relative General Plane Motion—Translating Axes

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$$

Relative General Plane Motion—Trans. and Rot. Axis

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

KINETICS

Mass Moment of Inertia

$$I = \int r^2 dm$$

Parallel-Axis Theorem

$$I = I_G + md^2$$

Radius of Gyration

$$k = \sqrt{\frac{I}{m}}$$

Equations of Motion

Particle	$\Sigma \mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G \alpha$ or $\Sigma M_P = \Sigma (\mathcal{M}_k)_P$

Principle of Work and Energy

$$T_1 + U_{1-2} = T_2$$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force

$$U_F = \int F \cos \theta ds$$

Constant force

$$U_F = (F_c \cos \theta) \Delta s$$

Weight

$$U_W = -W \Delta y$$

Spring

$$U_s = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$$

Couple moment

$$U_M = M \Delta \theta$$

Power and Efficiency

$$P = \frac{dU}{dt} = \mathbf{F} \cdot \mathbf{v} \quad \epsilon = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_{\text{out}}}{U_{\text{in}}}$$

Conservation of Energy Theorem

$$T_1 + V_1 = T_2 + V_2$$

Potential Energy

$$V = V_g + V_e, \text{ where } V_g = \pm W y, V_e = +\frac{1}{2}ks^2$$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$$\Sigma(\text{sys. } m\mathbf{v})_1 = \Sigma(\text{sys. } m\mathbf{v})_2$$

Coefficient of Restitution

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$

Conservation of Angular Momentum

$$\Sigma(\text{sys. } \mathbf{H})_1 = \Sigma(\text{sys. } \mathbf{H})_2$$

APPENDIX

A

Mathematical Expressions

Quadratic Formula

$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \cosh x = \frac{e^x + e^{-x}}{2}, \tanh x = \frac{\sinh x}{\cosh x}$$

Trigonometric Identities

$$\sin \theta = \frac{A}{C}, \csc \theta = \frac{C}{A}$$

$$\cos \theta = \frac{B}{C}, \sec \theta = \frac{C}{B}$$

$$\tan \theta = \frac{A}{B}, \cot \theta = \frac{B}{A}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos \theta = \pm \sqrt{\frac{1 + \cos 2\theta}{2}}, \sin \theta = \pm \sqrt{\frac{1 - \cos 2\theta}{2}}$$

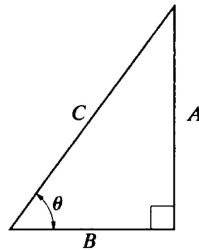
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

Power-Series Expansions

$$\sin x = x - \frac{x^3}{3!} + \dots \quad \sinh x = x + \frac{x^3}{3!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \dots \quad \cosh x = 1 + \frac{x^2}{2!} + \dots$$

**Derivatives**

$$\frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \tan u \sec u \frac{du}{dx}$$

$$\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$$

$$\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$$

Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx) + C$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ba}} \ln \left[\frac{a+x\sqrt{-ab}}{a-x\sqrt{-ab}} \right] + C, ab < 0$$

$$\int \frac{x dx}{a+bx^2} = \frac{1}{2b} \ln(bx^2+a) + C,$$

$$\int \frac{x^2 dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b\sqrt{ab}} \tan^{-1} \frac{x\sqrt{ab}}{a} + C, ab > 0$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left[\frac{a+x}{a-x} \right] + C, a^2 > x^2$$

$$\int \sqrt{a+bx} dx = \frac{2}{3b} \sqrt{(a+bx)^3} + C$$

$$\int x\sqrt{a+bx} dx = \frac{-2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2} + C$$

$$\int x^2\sqrt{a+bx} dx = \frac{2(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}}{105b^3} + C$$

$$\int \sqrt{a^2-x^2} dx = \frac{1}{2} \left[x\sqrt{a^2-x^2} + a^2 \sin^{-1} \frac{x}{a} \right] + C, a > 0$$

$$\int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3} \sqrt{(x^2 \pm a^2)^3} + C$$

$$\int x^2\sqrt{a^2-x^2} dx = -\frac{x}{4}\sqrt{(a^2-x^2)^3} + \frac{a^2}{8} \left(x\sqrt{a^2-x^2} + a^2 \sin^{-1} \frac{x}{a} \right) + C, a > 0$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2}) \right] + C$$

$$\int x\sqrt{a^2-x^2} dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + C$$

$$\int x^2\sqrt{x^2 \pm a^2} dx = \frac{x}{4}\sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8} x\sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln(x + \sqrt{x^2 \pm a^2}) + C$$

$$\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b} + C$$

$$\int \frac{x dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} + C$$

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}} \ln \left[\sqrt{a+bx+cx^2} + x\sqrt{c} + \frac{b}{2\sqrt{c}} \right] + C, c > 0$$

$$= \frac{1}{\sqrt{-c}} \sin^{-1} \left(\frac{-2cx-b}{\sqrt{b^2-4ac}} \right) + C, c < 0$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int x \cos(ax) dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax) + C$$

$$\int x^2 \cos(ax) dx = \frac{2x}{a^2} \cos(ax) + \frac{a^2x^2-2}{a^3} \sin(ax) + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

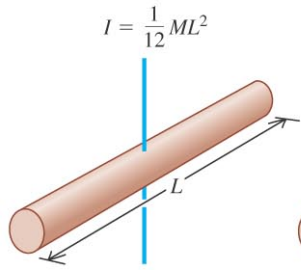
$$\int xe^{ax} dx = \frac{e^{ax}}{a^2} (ax-1) + C$$

$$\int \sinh x dx = \cosh x + C$$

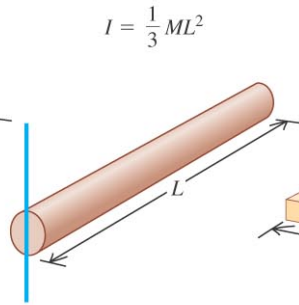
$$\int \cosh x dx = \sinh x + C$$

Table of Moment of Inertia

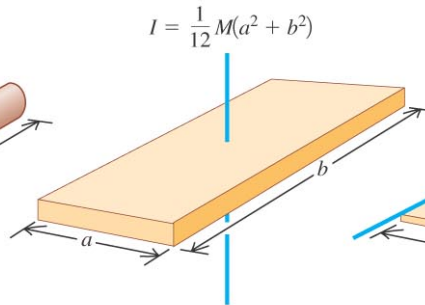
(a) Slender rod, axis through center



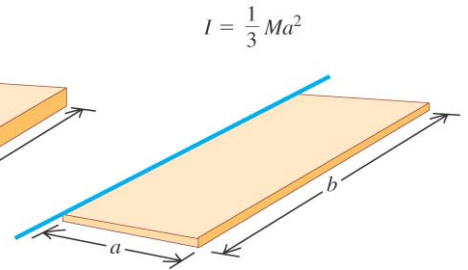
(b) Slender rod, axis through one end



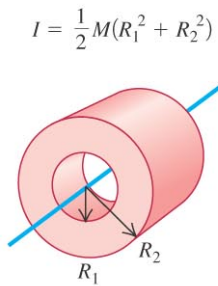
(c) Rectangular plate, axis through center



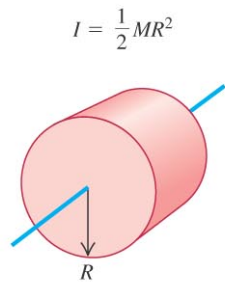
(d) Thin rectangular plate, axis along edge



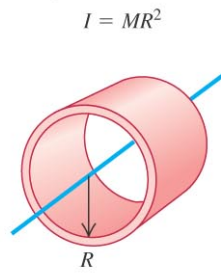
(e) Hollow cylinder



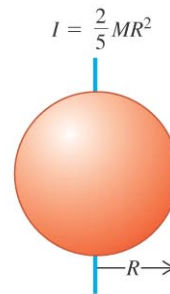
(f) Solid cylinder



(g) Thin-walled hollow cylinder



(h) Solid sphere



(i) Thin-walled hollow sphere

