Faculty of Engineering and Department of Physics

Engineering Physics 131

Midterm Examination

Monday February 26, 2018; 7:00 pm – 8:30 pm

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1. Closed book exam. No notes or textbooks allowed.
2. Formula sheets are included (may be removed).
3. The exam has **7** problems and is out of **50 points**. Attempt all parts of all problems.
4. *Questions 1 to 3 do not require detailed calculations and only the final answers to these questions will be marked.*
5. *For Questions 4 to 7, details and procedures to solve these problems will be marked.* Show all work in a neat and logical manner.
6. Write your solution directly on the pages with the questions. Indicate clearly if you use the backs of pages for material to be marked.
7. Only non-programmable calculator approved by the Faculty of Engineering permitted. Turn off all cell-phones, laptops, etc.

DO NOT separate the pages of the exam containing the problems.

LAST NAME: ____

FIRST NAME: ____

ID#: ____________________

Please circle the name of your instructor:

EB01: Wheelock

EB02: Jung

EB03: Wang

EB04: Kim

EB05: Gingrich

EB06: Tang

Address all inquiries to a supervisor. Do not communicate with other candidates. If you become ill during the exam, contact a supervisor immediately. (You may not claim extenuating circumstances and request your paper to be cancelled after writing and handing in your examination.) You may not leave the exam until at least 30 minutes have elapsed

End of Exam: When the signal is given to end the exam, students must promptly cease writing. If a student does not stop at the signal, the instructor has the discretion either not to grade the exam paper or to lower the grade on the examination.

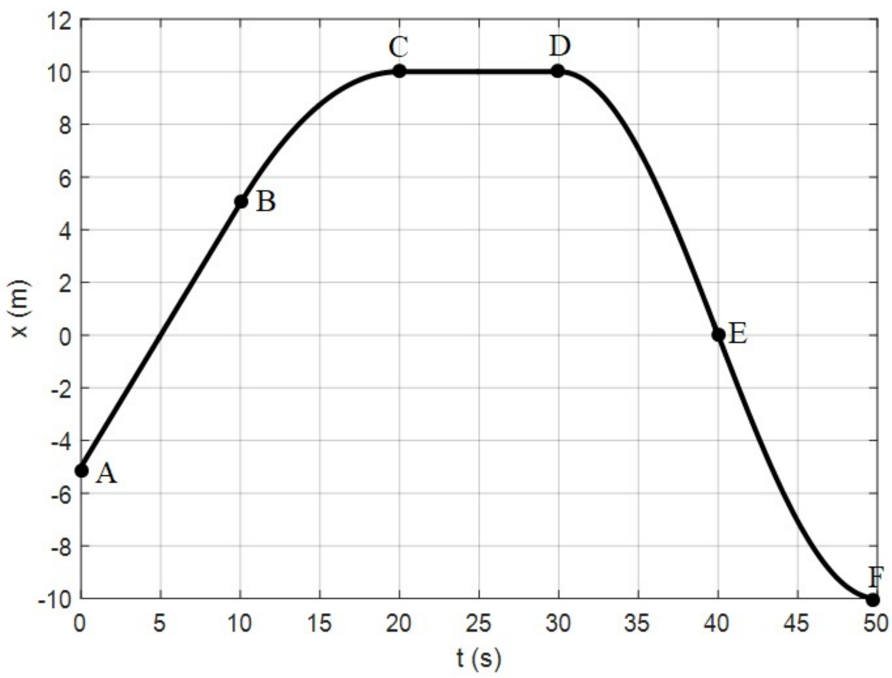
Please do not write in the table below.

Question	**Value (Points)**	**Mark**
1	3	
2	4	
3	5	
4	8	
5	8	
6	12	
7	10	
Total	50	

1. [3 Points]

The position of a particle traveling along the x-axis is given below as a function of time. The curve is straight between A and B, as well as between C and D. Point E is an inflection point.

[0.5 marks for each question, all correct answers must be selected in order to receive the 0.5 marks, no partial marks]



1) Circle the interval(s) in which the particle's velocity is positive

(A, B) (B, C) (C, D) (D, E) (E, F)

2) Circle the interval(s) in which the particle's velocity is negative

(A, B) (B, C) (C, D) (D, E) (E, F)

3) Circle the interval(s) in which the particle's velocity is zero

(A, B) (B, C) (C, D) (D, E) (E, F)

4) Circle the interval(s) in which the particle's acceleration is positive

(A, B) (B, C) (C, D) (D, E) (E, F)

5) Circle the interval(s) in which the particle's acceleration is negative

(A, B) (B, C) (C, D) (D, E) (E, F)

6) Circle the interval(s) in which the particle's acceleration is zero

(A, B) (B, C) (C, D) (D, E) (E, F)

Solution:

1) Circle the interval(s) in which the particle's velocity is positive

(A, B) (B, C) (C, D) (D, E) (E, F)

2) Circle the interval(s) in which the particle's velocity is negative

(A, B) (B, C) (C, D) (D, E) (E, F)

3) Circle the interval(s) in which the particle's velocity is zero

(A, B) (B, C) (C, D) (D, E) (E, F)

4) Circle the interval(s) in which the particle's acceleration is positive

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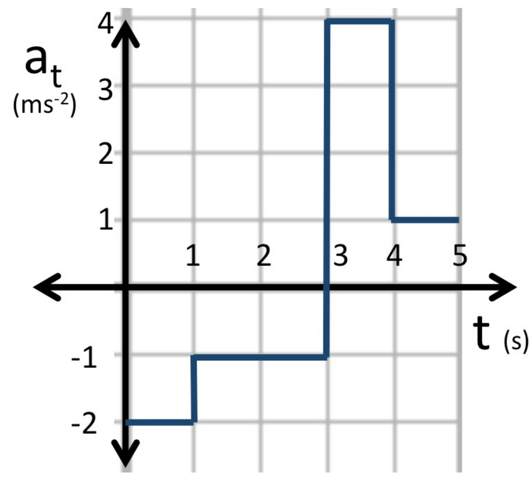
6) Circle the interval(s) in which the particle's acceleration is zero

(A, B) (B, C) (C, D) (D, E) (E, F)

2. [4 Points]

Tangential acceleration vs. time is plotted for a remote-controlled toy car traveling along a circular track with radius $r = 4\ m$. Assume that the car starts from rest at s_0 at $t = 0$.

For the following questions, **consider the time interval** $\boldsymbol{0 < t \leq 5s}$**.** You must include appropriate units or your answer will be marked wrong.



(a) [1 mark] At what time(s), if any, does the car reverse direction along its curvilinear path? Write "none" if it does not reverse direction.

___4 (s)____________________

(b) [1 mark] At what time(s), if any, does the car return to its initial position s_0? Write "none" if it does not return to s_0.

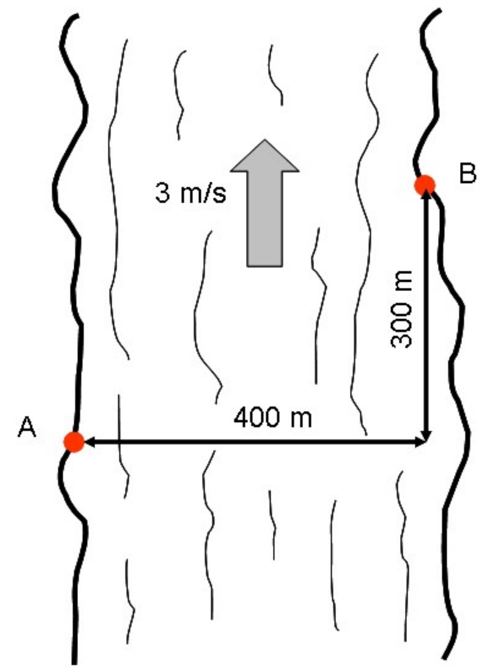
__none_____________________

(c) [2 mark] During this time interval the maximum value of the normal component of

acceleration is _4 (m/s$^{2)}$___________, which occurs at time t = _3 (s)__ .

3. [5 Points]

The river flows north at *3 m/s*. You are in a boat that moves at a constant speed of v relative to the water. You want to travel along a straight line from point A to point B by pointing the boat in a proper direction (this is the direction relative to the water). Consider the following situations:



(1) [1 mark] If $v = 5\,m/s$, which of the following corresponds to the direction in which you should point your boat, approximately?

(a) $\nearrow$ (b) $\longrightarrow$

(c) $\searrow$ (d) insufficient information

(2) [1 mark] If $v = 4\,m/s$, which of the following corresponds to the direction in which you should point your boat, approximately?

(a) $\nearrow$ (b) $\longrightarrow$

(c) $\searrow$ (d) insufficient information

(3) [1 mark] If $v = 3\,m/s$, which of the following corresponds to the direction in which you should point your boat, approximately?

(a) $\nearrow$ (b) $\longrightarrow$

(c) $\searrow$ (d) insufficient information

(4) [1 mark] What is the minimum value of v that will allow your boat to travel along a straight line from point A to point B?

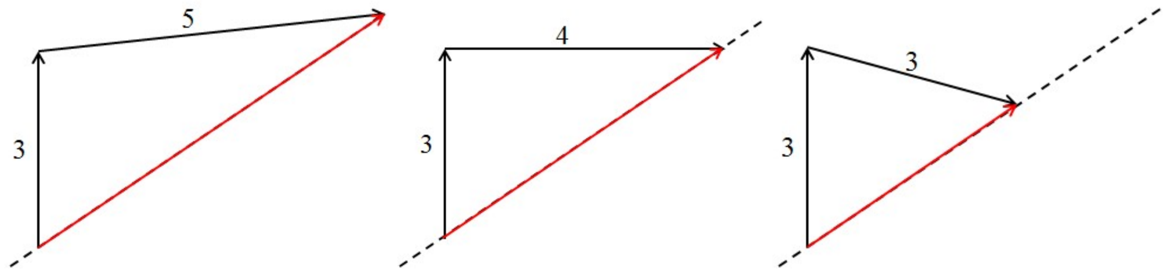
ANS: ____

(5) [1 mark] In the situation in (4), in which direction should you point the boat? Please answer the question by specifying the angle your boat makes with the direction of the river flow.

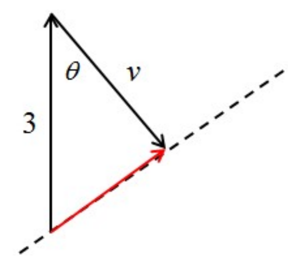
ANS: ____

Solution:

Velocity triangles for the three cases:



With the minimum value of v, the velocity triangle is:



$$\tan\theta = \frac{3}{4} \quad \Rightarrow \quad \cos\theta = \frac{4}{5} = \frac{v}{3} \quad \Rightarrow \quad v = 2.4 \text{ m/s}$$

The angle **v** makes with the direction of the river flow is:

$180^o - \theta = 143^o$

4. [8 Points]

The acceleration of a particle which is moving in the positive x direction is given by $a = \frac{kv^2}{x^3}$, where a is in m/s^2, v is in m/s and x is in m. The numerical value of the constant k is 2. The initial conditions at time $t = 0$ are $x_0 = 1$ m and $v_0 = 10$ m/s. Determine:

(1) [2 marks] the SI units of the constant k;
(2) [6 marks] the velocity of the particle when $x = 5$ m.

Solution:

Dimension homogeneity: $\frac{L}{T^2} = [k]\frac{(L/T)^2}{L^3}$ $\Rightarrow$ $[k] = L^2$, i.e., SI unit for k is m^2.

Use $adx = vdv$ $\Rightarrow$ $\frac{2v^2}{x^3}dx = vdv$

$$\Rightarrow \quad 2\frac{dx}{x^3} = \frac{dv}{v}$$

$$\Rightarrow \quad 2\int_1^x \frac{dx}{x^3} = \int_{10}^v \frac{dv}{v}$$

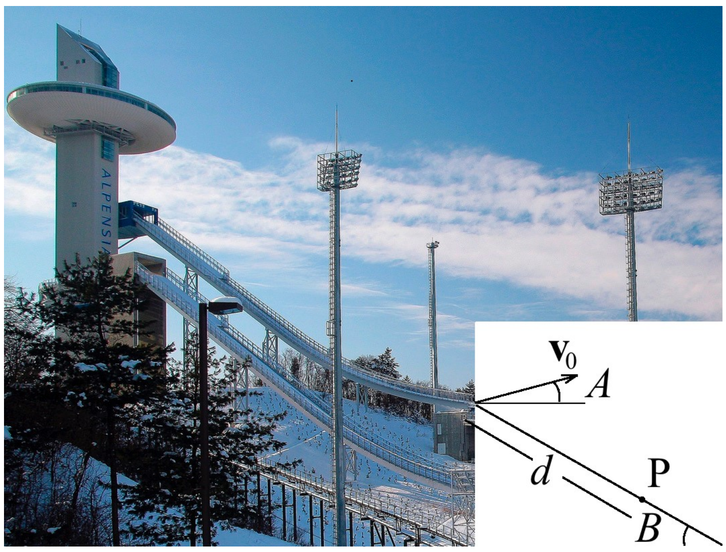
$$\Rightarrow \quad -\frac{1}{x^2}\bigg|_1^x = \ln|v|\Big|_{10}^v$$

$\Rightarrow \quad 1 - \frac{1}{x^2} = \ln\left(\frac{v}{10}\right)$ (v does not change sign) $\Rightarrow \quad v = 10\exp\left(1 - \frac{1}{x^2}\right)$

At $x = 5$ m, $\Rightarrow$ $v = 26.1$ m/s.

5. [8 Points]

During the 2018 Winter Olympics in PyeongChang, a skier leaves the ground with speed v_0 at an angle $A = 8.5°$ above the horizontal, as shown below. If she lands at P, a distance $d = 250\ ft$ down the slope (which we model as a straight line inclined at $B = 28\,°$), what is the initial speed v_0, in miles per hour ? (*1 mile = 5280 ft*)



Solution : We choose the origin at the position of departure, with the x-axis pointing to the right and the y-axis upward. Then the position varies with time as

$$x(t) = v_0(\cos A)t \tag{1}$$

and

$$y(t) = v_0(\sin A)t - \frac{1}{2}gt^2. \tag{2}$$

At time t', when the skier lands at a distance d down the slope, her position is given by

$$x(t') = d\cos B, \qquad y(t') = -d\sin B. \tag{3}$$

From Eqs. (1) and (3), we obtain the duration of flight,

$$t' = \frac{d\cos B}{v_0\cos A}.$$

When we substitute this into Eq. (2), together with Eq. (3), we obtain

$$-d\sin B = v_0 \sin A \frac{d\cos B}{v_0\cos A} - \frac{1}{2}g\frac{d^2\cos^2 B}{v_0^2\cos^2 A},$$

which we solve for v_0,

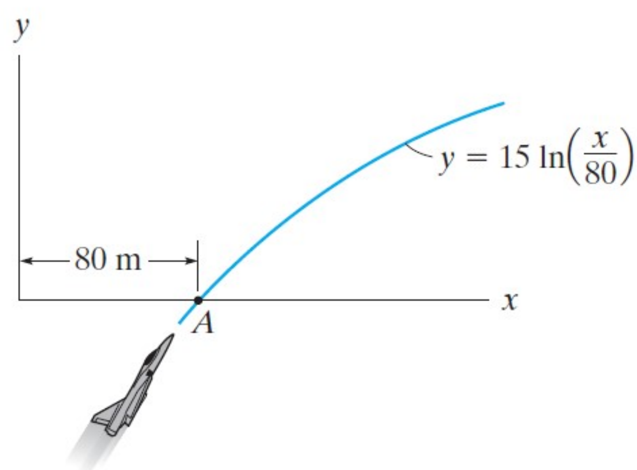
$$v_0 = \frac{\cos B}{\cos A}\sqrt{\frac{gd}{2\,(\sin B + \cos B\tan A)}} = \frac{\cos 28°}{\cos 8.5°}\sqrt{\frac{(32.2)(250)}{2\,(\sin 28° + \cos 28° \tan 8.5°)}} = 73\ \text{ft/s}$$

so that $v_0 = 73\ \text{ft/s} \times \frac{1\ \text{mile}}{5280\ \text{ft}} \times \frac{3600\ \text{s}}{1\ \text{h}} = \boxed{50\ \text{mph}}$

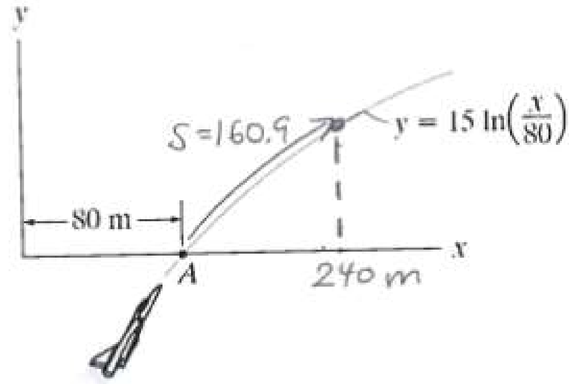
6. [12 Points]

The jet is travelling with a speed of *110 m/s* along the path shown below. When the jet reaches point A it begins to accelerate tangentially along the path at a rate of $0.2s\ m/s^2$, where s is the distance travelled along the path from point A.

(a) Determine the speed of the jet when it reaches $x = 240\ m$ $(s=160.9\ m)$.
(b) Determine the magnitude of the acceleration of the jet when it reaches $x = 240\ m$ $(s = 160.9\ m)$. Specify the angle this acceleration makes with respect to the positive x-axis.



Solution



(a) $v\,dv = a_t\,ds = 0.2\,s\,ds$

$$\int_{110}^{v} v\,dv = 0.2\int_0^{160.9} s\,ds \Rightarrow \frac{1}{2}v^2\Big|_{110}^{v} = 0.2\,\frac{s^2}{2}\Big|_0^{160.9}$$

$$\frac{1}{2}\left(v^2 - (110)^2\right) = \frac{1}{2}\left(0.2(160.9)^2\right) \Rightarrow \underline{v = 131.4 \text{ m/s}}$$

@ $x = 240$ m ($s = 160.9$ m)

(b) a @ $x = 240$ m ($s = 160.9$ m)

$$a_t = 0.2\,s = 0.2(160.9 \text{ m}) = 32.18 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}\ ; \quad v = 131.4 \text{ m/s}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\ ; \quad y = 15\ln\left(\frac{x}{80}\right);\ \frac{dy}{dx} = \frac{15}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2} \Rightarrow \left(\frac{dy}{dx}\right)_{x=240\text{m}} = 0.0625\ ; \quad \left(\frac{d^2y}{dx^2}\right)_{x=240\text{m}} = -2.604\times10^{-4}\ \frac{1}{\text{m}}$$

$$\rho = \frac{\left[1 + (0.0625)^2\right]^{3/2}}{\left|-2.604\times10^{-4}\ 1/\text{m}\right|} = 3863 \text{ m}$$

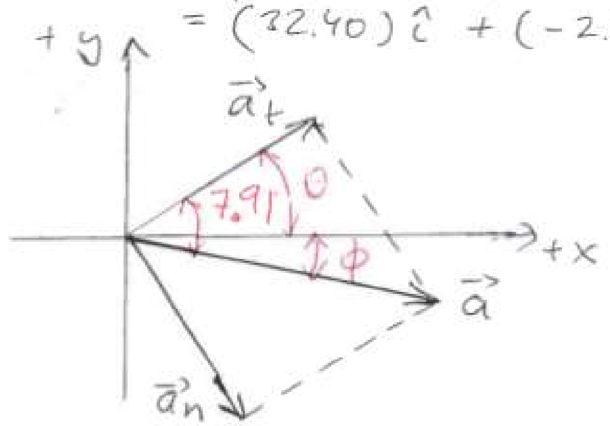
$$a_n = \frac{v^2}{\rho} = \frac{(131.4 \text{ m/s})^2}{3863 \text{ m}} = 4.473 \text{ m/s}^2$$

$$|a| = \left(a_n^2 + a_t^2\right)^{1/2} = 32.5 \text{ m/s}^2$$

$$\vec{a} = \left(a_t\cos\theta + a_n\sin\theta\right)\hat{i} + \left(a_t\sin\theta - a_n\cos\theta\right)\hat{j}$$

$$= (32.40)\,\hat{i} + (-2.457)\,\hat{j} \Rightarrow \phi = \tan^{-1}\left(\frac{-2.457}{32.40}\right)$$

$$\phi = -4.34°$$



$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)_{x=240\text{m}} = \tan^{-1}(0.0625) = 3.5763°$$

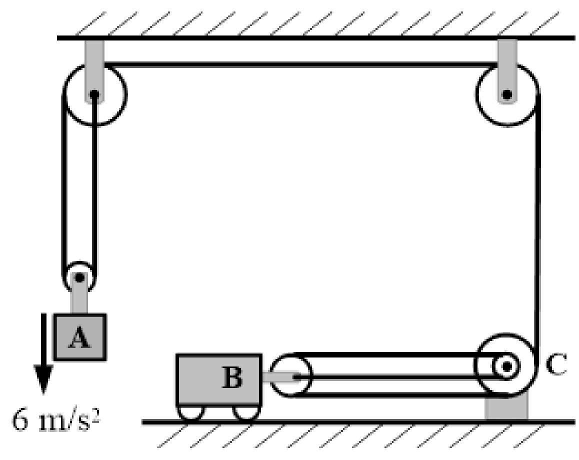
8

7. [10 Points]

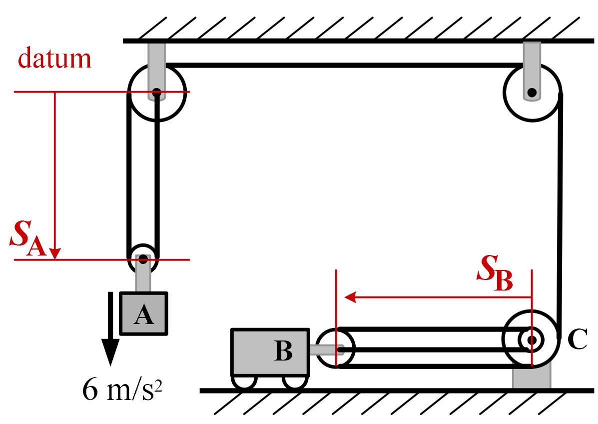
The pulley system is initially at rest (t = 0 s). If block ***A*** moves downward with a constant acceleration of $6\ m/s^2$:

a) Determine the velocity of ***B*** at time $t=3\ s$. Clearly specify both the magnitude and the direction.

b) Determine the time elapsed when the magnitude of relative velocity of ***B*** with respect to ***A*** becomes $25\ m/s$. Clearly specify the direction of this relative velocity and graphically show the direction.



Solution



a)

$2\boldsymbol{s}_A + 3\boldsymbol{s}_B = l$

take time derivative twice and get

$2\boldsymbol{a}_A + 3\boldsymbol{a}_B = 0$

$\boldsymbol{a}_B = -\frac{2}{3}\boldsymbol{a}_A$

Therefore

$\boldsymbol{a}_A = 6m/s^2 \downarrow$

$\boldsymbol{a}_B = 4m/s^2 \rightarrow$

$\boldsymbol{v}_B = \boldsymbol{a}_B t = 4t$

Therefore at t=3 s, we have

$\boldsymbol{v}_B = 12\left(\frac{m}{s}\right) \rightarrow$

b)

Since $adt = vdv$ and $(\boldsymbol{v}_A)_o = (\boldsymbol{v}_B)_o = 0$, we obtain

$\int_0^t \boldsymbol{a}_A dt = \int_0^{\boldsymbol{v}_A} dv \rightarrow \boldsymbol{v}_A = 6t \downarrow$

Similarly for $\boldsymbol{v}_B$

$\boldsymbol{v}_B = 4t \rightarrow$

Relative velocity is given by

$\boldsymbol{v}_{B/A} = \boldsymbol{v}_B - \boldsymbol{v}_A$ the we have

$\boldsymbol{v}_{B/A} = 4t\boldsymbol{i} - (-6t)\boldsymbol{j} = 4t\boldsymbol{i} + 6t\boldsymbol{j}$ (m/s)

Therefore, magnitude can be evaluated as

$|\boldsymbol{v}_{B/A}| = t\sqrt{4^2 + 6^2} = 7.21\text{t} = 25$

$t = 3.467s$