Chapitre 21 – Charges et champs électriques [9 au 12 mai]

DEVOIR : 21.78, 21.84, 21.56, 21.90, 21.71

21.1. Charge électrique et structure de la matière

- À lire rapidement. Concepts déjà familiers.

21.2. Conducteurs, isolants, charges induites

- Surtout *Charging by Induction* et *Electric Forces on Uncharged Objects*

21.3. Loi de Coulomb

- Eq. (21.2)
- P. 719 *Superposition of Forces*
- Lire tous les exemples
- Autres exemples:

21.10. **(a) IDENTIFY:** The electrical attraction of the proton gives the electron an acceleration equal to the acceleration due to gravity on earth.

SET UP: Coulomb's law gives the force and Newton's second law gives the acceleration this force produces. $ma = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_2|}{r^2}$ and $r = \sqrt{\dfrac{e^2}{4\pi\epsilon_0 ma}}$.

EXECUTE: $r = \sqrt{\dfrac{\left(9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}} = 5.08\ \text{m}$

EVALUATE: The electron needs to be about 5 m from a single proton to have the same acceleration as it receives from the gravity of the entire earth.

(b) IDENTIFY: The force on the electron comes from the electrical attraction of all the protons in the earth.

SET UP: First find the number n of protons in the earth, and then find the acceleration of the electron using Newton's second law, as in part (a).

$$n = m_E/m_p = (5.97\times10^{24}\ \text{kg})/(1.67\times10^{-27}\ \text{kg}) = 3.57\times10^{51}\ \text{protons.}$$

$$a = F/m = \frac{\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_p q_e|}{R_E^2}}{m_e} = \frac{\dfrac{1}{4\pi\epsilon_0}ne^2}{m_e R_E^2}.$$

EXECUTE: $a = (9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.57\times10^{51})(1.60\times10^{-19}\ \text{C})^2/[(9.11\times10^{-31}\ \text{kg})(6.38\times10^{6}\ \text{m})^2] = 2.22\times10^{40}\ \text{m/s}^2$. One can ignore the gravitation force since it produces an acceleration of only $9.8\ \text{m/s}^2$ and hence is much much less than the electrical force.

EVALUATE: With the electrical force, the acceleration of the electron would nearly 10^{40} times greater than with gravity, which shows how strong the electrical force is.

21.16. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_2.

SET UP: $\vec{F}_{2\text{ on }1}$ is in the $+y$-direction.

EXECUTE: $F_{2\text{on }1} = \dfrac{(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\,(2.0\times10^{-6}\ \text{C})\,(2.0\times10^{-6}\ \text{C})}{(0.60\ \text{m})^2} = 0.100\ \text{N}$. $\left(F_{2\text{ on }1}\right)_x = 0$ and $\left(F_{2\text{ on }1}\right)_y = +0.100\ \text{N}$. $F_{Q\text{ on }1}$ is equal and opposite to $F_{1\text{ on }Q}$ (Example 21.4), so $\left(F_{Q\text{ on }1}\right)_x = -0.23\text{N}$ and $\left(F_{Q\text{ on }1}\right)_y = 0.17\ \text{N}$. $F_x = \left(F_{2\text{ on }1}\right)_x + \left(F_{Q\text{ on }1}\right)_x = -0.23\ \text{N}$.

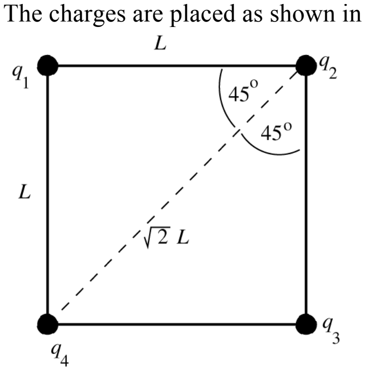
$F_y = \left(F_{2\text{ on }1}\right)_y + \left(F_{Q\text{ on }1}\right)_y = 0.100\ \text{N} + 0.17\ \text{N} = 0.27\ \text{N}$. The magnitude of the total force is

$F = \sqrt{(0.23\ \text{N})^2 + (0.27\ \text{N})^2} = 0.35\ \text{N}$. $\tan^{-1}\dfrac{0.23}{0.27} = 40°$, so $\vec{F}$ is $40°$ counterclockwise from the $+y$ axis, or $130°$ counterclockwise from the $+x$ axis.

EVALUATE: Both forces on q_1 are repulsive and are directed away from the charges that exert them.

21.23. **IDENTIFY:** Apply Coulomb's law to calculate the force exerted on one of the charges by each of the other three and then add these forces as vectors.

(a) SET UP: The charges are placed as shown in Figure 21.23a.



$q_1 = q_2 = q_3 = q_4 = q$

Figure 21.23a

Consider forces on q_4. The free-body diagram is given in Figure 21.23b. Take the y-axis to be parallel to the diagonal between q_2 and q_4 and let $+y$ be in the direction away from q_2. Then $\vec{F}_2$ is in the $+y$ -direction.

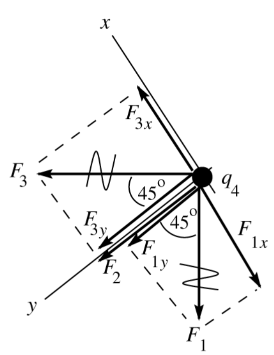


Figure 21.23b

EXECUTE: $F_3 = F_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{L^2}$

$F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{2L^2}$

$F_{1x} = -F_1\sin45° = -F_1/\sqrt{2}$

$F_{1y} = +F_1\cos45° = +F_1/\sqrt{2}$

$F_{3x} = +F_3\sin45° = +F_3/\sqrt{2}$

$F_{3y} = +F_3\cos45° = +F_3/\sqrt{2}$

$F_{2x} = 0,\ F_{2y} = F_2$

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 0$

$$R_y = F_{1y} + F_{2y} + F_{3y} = (2/\sqrt{2})\frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2} + \frac{1}{4\pi\epsilon_0}\frac{q^2}{2L^2} = \frac{q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2})$$

$$R = \frac{q^2}{8\pi\epsilon_0 L^2}(1 + 2\sqrt{2}).$$ Same for all four charges.

EVALUATE: In general the resultant force on one of the charges is directed away from the opposite corner. The forces are all repulsive since the charges are all the same. By symmetry the net force on one charge can have no component perpendicular to the diagonal of the square.

21.24. **IDENTIFY:** Apply $F = \frac{k|qq'|}{r^2}$ to find the force of each charge on $+q$. The net force is the vector sum of the individual forces.

SET UP: Let $q_1 = +2.50\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge q_1, since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of q_1. Let $+q$ be a distance d to the left of q_1, so it is a distance $d + 0.600\ \text{m}$ from q_2.

EXECUTE: $F_1 = F_2$ gives $\frac{kq|q_1|}{d^2} = \frac{kq|q_2|}{(d + 0.600\ \text{m})^2}$.

$d = \pm\sqrt{\frac{|q_1|}{|q_2|}}(d + 0.600\ \text{m}) = \pm(0.8452)(d + 0.600\ \text{m})$. d must be positive, so

$d = \frac{(0.8452)(0.600\ \text{m})}{1 - 0.8452} = 3.27\ \text{m}$. The net force would be zero when $+q$ is at $x = -3.27\ \text{m}$.

EVALUATE: When $+q$ is at $x = -3.27\ \text{m}$, $\vec{F}_1$ is in the $-x$ direction and $\vec{F}_2$ is in the $+x$ direction.

21.4. Champs électriques et forces électriques

- Eq. (21.3) est la définition de **E**. En principe, on utilise plutôt Eq. (21.4), car étant donné un champ **E**, si on y place une charge q_0, elle subira une force donnée par Eq. (21.4).
- En général, la force obtenue de Eq. (21.4) *n'est pas* la loi de Coulomb, Eq. (21.2).
- Le cas particulier d'une *charge source ponctuelle* est donné par Eq. (21.7).
- Lire tous les exemples.
- Autres exemples:

21.28. **IDENTIFY:** Use constant acceleration equations to calculate the upward acceleration a and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let $+y$ be upward. An electron has charge $q = -e$.

EXECUTE: **(a)** $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$$a = \frac{2(y - y_0)}{t^2} = \frac{2(4.50\ \text{m})}{(3.00 \times 10^{-6}\ \text{s})^2} = 1.00 \times 10^{12}\ \text{m/s}^2.$$

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11 \times 10^{-31}\ \text{kg})(1.00 \times 10^{12}\ \text{m/s}^2)}{1.60 \times 10^{-19}\ \text{C}} = 5.69\ \text{N/C}$$

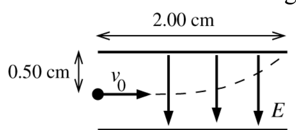
The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim 10^{11} g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.33. **IDENTIFY:** Eq. (21.3) gives the force on the particle in terms of its charge and the electric field between the plates. The force is constant and produces a constant acceleration. The motion is similar to projectile motion; use constant acceleration equations for the horizontal and vertical components of the motion.

(a) SET UP: The motion is sketched in Figure 21.33a.



For an electron $q = -e$.

Figure 21.33a

$\vec{F} = q\vec{E}$ and q negative gives that $\vec{F}$ and $\vec{E}$ are in opposite directions, so $\vec{F}$ is upward. The free-body diagram for the electron is given in Figure 21.33b.

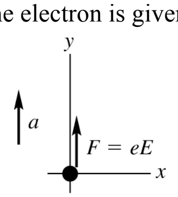


Figure 21.33b

EXECUTE: $\sum F_y = ma_y$

$eE = ma$

Solve the kinematics to find the acceleration of the electron: Just misses upper plate says that $x - x_0 = 2.00$ cm when $y - y_0 = +0.500$ cm.

<u>x-component</u>

$v_{0x} = v_0 = 1.60 \times 10^6$ m/s, $a_x = 0$, $x - x_0 = 0.0200$ m, $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.0200 \text{ m}}{1.60 \times 10^6 \text{ m/s}} = 1.25 \times 10^{-8} \text{ s}$$

In this same time t the electron travels 0.0050 m vertically:

<u>y-component</u>

$t = 1.25 \times 10^{-8}$ s, $v_{0y} = 0$, $y - y_0 = +0.0050$ m, $a_y = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.0050 \text{ m})}{(1.25 \times 10^{-8} \text{ s})^2} = 6.40 \times 10^{13} \text{ m/s}^2$$

(This analysis is very similar to that used in Chapter 3 for projectile motion, except that here the acceleration is upward rather than downward.) This acceleration must be produced by the electric-field force: $eE = ma$

$$E = \frac{ma}{e} = \frac{(9.109 \times 10^{-31} \text{ kg})(6.40 \times 10^{13} \text{ m/s}^2)}{1.602 \times 10^{-19} \text{ C}} = 364 \text{ N/C}$$

Note that the acceleration produced by the electric field is <u>much</u> larger than g, the acceleration produced by gravity, so it is perfectly ok to neglect the gravity force on the elctron in this problem.

(b) $a = \dfrac{eE}{m_p} = \dfrac{(1.602 \times 10^{-19} \text{ C})(364 \text{ N/C})}{1.673 \times 10^{-27} \text{ kg}} = 3.49 \times 10^{10} \text{ m/s}^2$

This is much less than the acceleration of the electron in part (a) so the vertical deflection is less and the proton won't hit the plates. The proton has the same initial speed, so the proton takes the same time $t = 1.25\times10^{-8}$ s to travel horizontally the length of the plates. The force on the proton is downward (in the
same direction as $\vec{E}$, since q is positive), so the acceleration is downward and $a_y = -3.49\times10^{10}\ \text{m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.49\times10^{10}\ \text{m/s}^2)(1.25\times10^{-8}\ \text{s})^2 = -2.73\times10^{-6}\ \text{m}$. The displacement is 2.73×10^{-6} m, downward.

(c) EVALUATE: The displacements are in opposite directions because the electron has negative charge and the proton has positive charge. The electron and proton have the same magnitude of charge, so the force the electric field exerts has the same magnitude for each charge. But the proton has a mass larger by a factor of 1836 so its acceleration and its vertical displacement are smaller by this factor.

21.40. IDENTIFY: The net force on each charge must be zero.

SET UP: The force diagram for the $-6.50\ \mu\text{C}$ charge is given in Figure 21.40. F_E is the force exerted on the charge by the uniform electric field. The charge is negative and the field is to the right, so the force exerted by the field is to the left. F_q is the force exerted by the other point charge. The two charges have opposite signs, so the force is attractive. Take the $+x$ axis to be to the right, as shown in the figure.

EXECUTE: **(a)** $F = |q|E = (6.50\times10^{-6}\ \text{C})(1.85\times10^{8}\ \text{N/C}) = 1.20\times10^{3}\ \text{N}$

$$F_q = k\frac{|q_1q_2|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6.50\times10^{-6}\ \text{C})(8.75\times10^{-6}\ \text{C})}{(0.0250\ \text{m})^2} = 8.18\times10^2\ \text{N}$$

$\sum F_x = 0$ gives $T + F_q - F_E = 0$ and $T = F_E - F_q = 382\ \text{N}$.

(b) Now F_q is to the left, since like charges repel.

$\sum F_x = 0$ gives $T - F_q - F_E = 0$ and $T = F_E + F_q = 2.02\times10^3\ \text{N}$.

EVALUATE: The tension is much larger when both charges have the same sign, so the force one charge exerts on the other is repulsive.

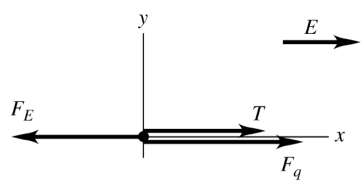


Figure 21.40

21.5. Champs électriques: points, dipôles, lignes chargées, sphères, disques,…

- Section très pratique. Le principe fondamental est la superposition. Lire les exemples des pp. 728-733.
- Autres exemples:

21.44. IDENTIFY: For a point charge, $E = k\frac{|q|}{r^2}$. For the net electric field to be zero, $\vec{E}_1$ and $\vec{E}_2$ must have equal magnitudes and opposite directions.

SET UP: Let $q_1 = +0.500$ nC and $q_2 = +8.00$ nC. $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: The two charges and the directions of their electric fields in three regions are shown in Figure 21.44. Only in region II are the two electric fields in opposite directions. Consider a point a distance x from q_1 so a distance $1.20\text{ m} - x$ from q_2. $E_1 = E_2$ gives $k\dfrac{0.500\text{ nC}}{x^2} = k\dfrac{8.00\text{ nC}}{(1.20 - x)^2}$.

$16x^2 = (1.20\text{ m} - x)^2$. $4x = \pm(1.20\text{ m} - x)$ and $x = 0.24\text{ m}$ is the positive solution. The electric field is zero at a point between the two charges, 0.24 m from the 0.500 nC charge and 0.96 m from the 8.00 nC charge.

EVALUATE: There is only one point along the line connecting the two charges where the net electric field is zero. This point is closer to the charge that has the smaller magnitude.

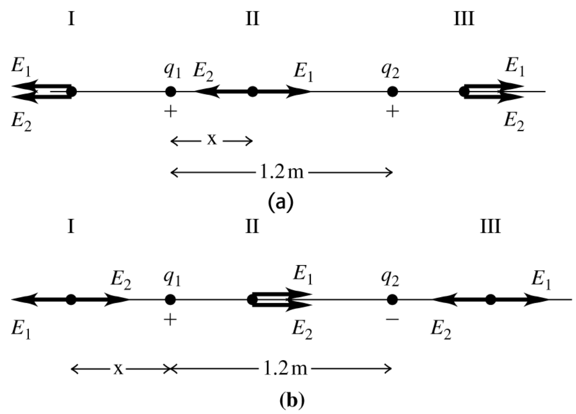


Figure 21.44

21.48. **IDENTIFY:** A positive and negative charge, of equal magnitude q, are on the x-axis, a distance a from the origin. Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of these fields.

SET UP: $\vec{E}$ due to a point charge is directed away from the charge if it is positive and directed toward the charge if it is negative.

EXECUTE: **(a)** Halfway between the charges, both fields are in the $-x$-direction and $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{a^2}$, in the $-x$-direction .

(b) $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right)$ for $|x| < a$. $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right)$ for $x > a$.

$E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right)$ for $x < -a$. E_x is graphed in Figure 21.48.

EVALUATE: At points on the x axis and between the charges, E_x is in the $-x$-direction because the fields from both charges are in this direction. For $x < -a$ and $x > +a$, the fields from the two charges are in opposite directions and the field from the closer charge is larger in magnitude.

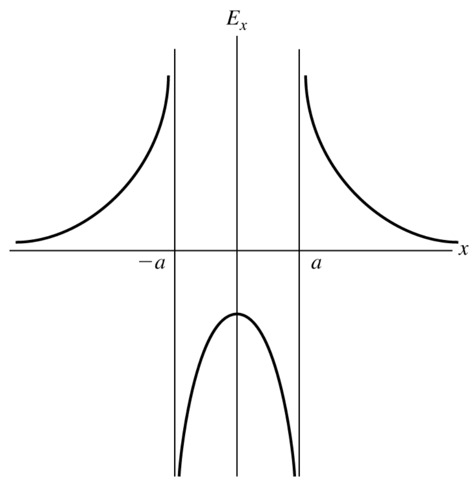


Figure 21.48

21.54. **(a) IDENTIFY:** The field is caused by a finite uniformly charged wire.

SET UP: The field for such a wire a distance x from its midpoint is

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x/a)^2+1}} = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda}{x\sqrt{(x/a)^2+1}}.$$

EXECUTE: $E = \dfrac{\left(18.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(175\times10^{-9}\ \text{C/m}\right)}{(0.0600\ \text{m})\sqrt{\left(\dfrac{6.00\ \text{cm}}{4.25\ \text{cm}}\right)^2+1}}$ = 3.03 × 10^4 N/C, directed upward.

(b) IDENTIFY: The field is caused by a uniformly charged circular wire.

SET UP: The field for such a wire a distance x from its midpoint is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. We first find the radius of the circle using $2\pi r = l$.

EXECUTE: Solving for r gives $r = l/2\pi$ = (8.50 cm)/2π = 1.353 cm

The charge on this circle is $Q = \lambda l$ = (175 nC/m)(0.0850 m) = 14.88 nC

The electric field is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}} = \frac{\left(9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(14.88\times10^{-9}\,\text{C/m}\right)\left(0.0600\ \text{m}\right)}{\left[(0.0600\ \text{m})^2+(0.01353\ \text{m})^2\right]^{3/2}}$$

E = 3.45 × 10^4 N/C, upward.

EVALUATE: In both cases, the fields are of the same order of magnitude, but the values are different because the charge has been bent into different shapes.

21.6. Lignes de champ électrique

- À lire rapidement

21.7. Dipôles électriques

- Un dipôle est tout simplement une paire de charges de même grandeur et de signes opposés.

- Le vecteur “moment dipolaire électrique” (ou *electric dipole moment*) a la grandeur donnée par Eq. (21.14) et la direction pointe de la charge – vers la charge +. Unités : C·m.
- Moment de force de **E** sur **p** : Eq. (21.16). Grandeur par Eq. (21.15)
- Énergie potentielle de **p** dans **E** : Eq. (21.18). Grandeur par Eq. (21.17). L’angle ϕ est l’angle entre **p** et **E**, $\phi = 0$ est la position où **p** est parallèle à **E**.
- Lire l’exemple de la p. 738
- Autres exemples :

21.65. **IDENTIFY:** Follow the procedure specified in part (a) of the problem.
SET UP: Use that $y >> d$.
EXECUTE: **(a)** $\frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2} = \frac{(y+d/2)^2-(y-d/2)^2}{(y^2-d^2/4)^2} = \frac{2yd}{(y^2-d^2/4)^2}$. This gives $E_y = \frac{q}{4\pi\epsilon_0}\frac{2yd}{(y^2-d^2/4)^2} = \frac{qd}{2\pi\epsilon_0}\frac{y}{(y^2-d^2/4)^2}$. Since $y^2 >> d^2/4$, $E_y \approx \frac{p}{2\pi\epsilon_0 y^3}$.

(b) For points on the $-y$-axis, $\vec{E}_-$ is in the $+y$ direction and $\vec{E}_+$ is in the $-y$ direction. The field point is closer to $-q$, so the net field is upward. A similar derivation gives $E_y \approx \frac{p}{2\pi\epsilon_0 y^3}$. E_y has the same magnitude and direction at points where $y >> d$ as where $y << -d$.

EVALUATE: E falls off like $1/r^3$ for a dipole, which is faster than the $1/r^2$ for a point charge. The total charge of the dipole is zero.

21.67. **IDENTIFY:** Like charges repel and unlike charges attract. The force increases as the distance between the charges decreases.
SET UP: The forces on the dipole that is between the slanted dipoles are sketched in Figure 21.67a.
EXECUTE: The forces are attractive because the + and $-$ charges of the two dipoles are closest. The forces are toward the slanted dipoles so have a net upward component. In Figure 21.67b, adjacent dipoles charges of opposite sign are closer than charges of the same sign so the attractive forces are larger than the repulsive forces and the dipoles attract.
EVALUATE: Each dipole has zero net charge, but because of the charge separation there is a non-zero force between dipoles.

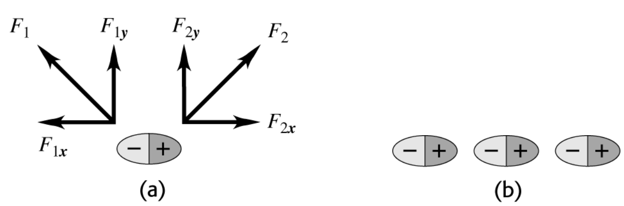


Figure 21.67

21.70. **IDENTIFY:** The plates produce a uniform electric field in the space between them. This field exerts torque on a dipole and gives it potential energy.
SET UP: The electric field between the plates is given by $E = \sigma/\epsilon_0$, and the dipole moment is $p = ed$. The potential energy of the dipole due to the field is $U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, and the torque the field exerts on it is $\tau = pE\sin\phi$.

EXECUTE: **(a)** The potential energy, $U = -\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{E}} = -pE\cos\phi$, is a maximum when $\phi = 180°$. The field between the plates is $E = \sigma / \epsilon_0$, giving

$$U_{\text{max}} = (1.60 \times 10^{-19}\ \text{C})(220 \times 10^{-9}\ \text{m})(125 \times 10^{-6}\ \text{C/m}^2)/(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2) = 4.97 \times 10^{-19}\ \text{J}$$

The orientation is parallel to the electric field (perpendicular to the plates) with the positive charge of the dipole toward the positive plate.

(b) The torque, $\tau = pE \sin \phi$, is a maximum when $\phi = 90°$ or $270°$. In this case

$$\tau_{\text{max}} = pE = p\sigma / \epsilon_0 = ed\sigma / \epsilon_0$$

$$\tau_{\text{max}} = \left(1.60 \times 10^{-19}\ \text{C}\right)\left(220 \times 10^{-9}\ \text{m}\right)\left(125 \times 10^{-6}\ \text{C/m}^2\right) \Big/ \left(8.85 \times 10^{-12}\ \text{C}^2 / \text{N} \cdot \text{m}^2\right)$$

$$\tau_{\text{max}} = 4.97 \times 10^{-19}\ \text{N} \cdot \text{m}$$

The dipole is oriented perpendicular to the electric field (parallel to the plates).

(c) $F = 0$.

EVALUATE: When the potential energy is a maximum, the torque is zero. In both cases, the net force on the dipole is zero because the forces on the charges are equal but opposite (which would not be true in a nonuniform electric field).