Chapitre 22 – Loi de Gauss [13 au 17 mai]

DEVOIR : 22.32, 22.34, 22.37, 22.42, 22.52

La loi de Gauss est l'une des quatre équations fondamentales de l'électromagnétisme, appelées "équations de Maxwell" (discutées à la section 29.7). Elle est donnée par l'Eq. (22.8)

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

où $\Phi_E = \oint \vec{E} \cdot d\vec{A}$ est le *flux électrique* au travers une surface fermée, et Q_{encl} est la charge totale contenue dans le volume délimité par la surface. En ce qui nous concerne, cette relation sera souvent utilisée non pas pour calculer Q_{encl}, mais pour trouver l'expression du champ pour des distributions de charges présentant certaines symétries.

22.1. Charge et flux électrique

- À lire rapidement

22.2. Calcul du flux électrique

- L'analogie des fluides aide à avoir une intuition du concept de flux. Le débit volumique $\vec{v} \cdot \vec{A}$ a la même forme que l'intégrant de l'Eq. (22.5).
- Lire les exemples pp 756 et 757.
- Autres exemples:

22.2. **IDENTIFY:** The field is uniform and the surface is flat, so use $\Phi_E = EA\cos\phi$.

SET UP: ϕ is the angle between the normal to the surface and the direction of $\vec{E}$, so $\phi = 70°$.

EXECUTE: $\Phi_E = (75.0\ \text{N/C})(0.400\ \text{m})(0.600\ \text{m})\cos 70° = 6.16\ \text{N}\cdot\text{m}^2/\text{C}$

EVALUATE: If the field were perpendicular to the surface the flux would be $\Phi_E = EA = 18.0\ \text{N}\cdot\text{m}^2/\text{C}$. The flux in this problem is much less than this because only the component of $\vec{E}$ perpendicular to the surface contributes to the flux.

22.4. **IDENTIFY:** Use Eq.(22.3) to calculate the flux for each surface. Use Eq.(22.8) to calculate the total enclosed charge.

SET UP: $\vec{E} = (-5.00\ \text{N/C}\cdot\text{m})x\hat{i} + (3.00\ \text{N/C}\cdot\text{m})z\hat{k}$. The area of each face is L^2, where $L = 0.300$ m.

EXECUTE: $\hat{n}_{S_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E}\cdot\hat{n}_{S_1}A = 0$.

$\hat{n}_{S_2} = +\hat{k} \Rightarrow \Phi_2 = \vec{E}\cdot\hat{n}_{S_2}A = (3.00\ \text{N/C}\cdot\text{m})(0.300\ \text{m})^2 z = (0.27\ (\text{N/C})\cdot\text{m})z$.

$\Phi_2 = (0.27\ (\text{N/C})\text{m})(0.300\ \text{m}) = 0.081\ (\text{N/C})\cdot\text{m}^2$.

$\hat{n}_{S_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E}\cdot\hat{n}_{S_3}A = 0$.

$\hat{n}_{S_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E}\cdot\hat{n}_{S_4}A = -(0.27\ (\text{N/C})\cdot\text{m})z = 0$ (since $z = 0$).

$\hat{n}_{S_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E}\cdot\hat{n}_{S_5}A = (-5.00\ \text{N/C}\cdot\text{m})(0.300\ \text{m})^2 x = -(0.45\ (\text{N/C})\cdot\text{m})x$.

$\Phi_5 = -(0.45\ (\text{N/C})\cdot\text{m})(0.300\ \text{m}) = -(0.135\ (\text{N/C})\cdot\text{m}^2)$.

$\hat{\boldsymbol{n}}_{S_6} = -\hat{\boldsymbol{i}} \Rightarrow \Phi_6 = \vec{\boldsymbol{E}} \cdot \hat{\boldsymbol{n}}_{S_6} A = +(0.45\ (\text{N/C}) \cdot \text{m})x = 0$ (since $x = 0$).

(b) Total flux: $\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)(\text{N/C}) \cdot \text{m}^2 = -0.054\ \text{N} \cdot \text{m}^2/\text{C}$. Therefore,

$q = P_0 \Phi = -4.78 \times 10^{-13}\ \text{C}$.

EVALUATE: Flux is positive when $\vec{\boldsymbol{E}}$ is directed out of the volume and negative when it is directed into the volume.

22.6. **IDENTIFY:** Use Eq.(22.3) to calculate the flux for each surface.

SET UP: $\Phi = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}} = EA\cos\phi$ where $\vec{\boldsymbol{A}} = A\hat{\boldsymbol{n}}$.

EXECUTE: **(a)** $\hat{\boldsymbol{n}}_{S_1} = -\hat{\boldsymbol{j}}$ (left). $\Phi_{S_1} = -(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos(90∞ - 36.9°) = -24\ \text{N} \cdot \text{m}^2/\text{C}$.

$\hat{\boldsymbol{n}}_{S_2} = +\hat{\boldsymbol{k}}$ (top). $\Phi_{S_2} = -(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos 90° = 0$.

$\hat{\boldsymbol{n}}_{S_3} = +\hat{\boldsymbol{j}}$ (right). $\Phi_{S_3} = +(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos(90° - 36.9°) = +24\ \text{N} \cdot \text{m}^2/\text{C}$.

$\hat{\boldsymbol{n}}_{S_4} = -\hat{\boldsymbol{k}}$ (bottom). $\Phi_{S_4} = (4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos 90° = 0$.

$\hat{\boldsymbol{n}}_{S_5} = +\hat{\boldsymbol{i}}$ (front). $\Phi_{S_5} = +(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos 36.9° = 32\ \text{N} \cdot \text{m}^2/\text{C}$.

$\hat{\boldsymbol{n}}_{S_6} = -\hat{\boldsymbol{i}}$ (back). $\Phi_{S_6} = -(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos 36.9° = -32\ \text{N} \cdot \text{m}^2/\text{C}$.

EVALUATE: **(b)** The total flux through the cube must be zero; any flux entering the cube must also leave it, since the field is uniform. Our calculation gives the result; the sum of the fluxes calculated in part (a) is zero.

22.3. Loi de Gauss

- Section centrale de ce chapitre: Eq. (22.8). Quoique plusieurs applications seront montrées à la section 22.4.
- Autres exemples :

22.8. **IDENTIFY:** Apply Gauss's law to each surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by each surface. Flux out of the volume is positive and flux into the enclosed volume is negative.

EXECUTE: **(a)** $\Phi_{S_1} = q_1/P_0 = (4.00 \times 10^{-9}\ \text{C})/P_0 = 452\ \text{N} \cdot \text{m}^2/\text{C}$.

(b) $\Phi_{S_2} = q_2/P_0 = (-7.80 \times 10^{-9}\ \text{C})/P_0 = -881\ \text{N} \cdot \text{m}^2/\text{C}$.

(c) $\Phi_{S_3} = (q_1 + q_2)/P_0 = ((4.00 - 7.80) \times 10^{-9}\ \text{C})/P_0 = -429\ \text{N} \cdot \text{m}^2/\text{C}$.

(d) $\Phi_{S_4} = (q_1 + q_3)/P_0 = ((4.00 + 2.40) \times 10^{-9}\ \text{C})/P_0 = 723\ \text{N} \cdot \text{m}^2/\text{C}$.

(e) $\Phi_{S_5} = (q_1 + q_2 + q_3)/P_0 = ((4.00 - 7.80 + 2.40) \times 10^{-9}\ \text{C})/P_0 = -158\ \text{N} \cdot \text{m}^2/\text{C}$.

EVALUATE: **(f)** All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.14. **IDENTIFY:** Apply the results of Examples 22.9 and 22.10.

SET UP: $E = k\dfrac{|q|}{r^2}$ outside the sphere. A proton has charge $+e$.

EXECUTE: **(a)** $E = k\dfrac{|q|}{r^2} = (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\dfrac{92(1.60 \times 10^{-19}\ \text{C})}{(7.4 \times 10^{-15}\ \text{m})^2} = 2.4 \times 10^{21}\ \text{N/C}$

(b) For $r = 1.0\times10^{-10}\ \text{m}$, $E = (2.4\times10^{21}\ \text{N/C})\left(\dfrac{7.4\times10^{-15}\ \text{m}}{1.0\times10^{-10}\ \text{m}}\right)^2 = 1.3\times10^{13}\ \text{N/C}$

(c) $E = 0$, inside a spherical shell.
EVALUATE: The electric field in an atom is very large.

22.4. Applications, symétries sphérique, cylindrique et planaire

- Quand une distribution présente une certaine symétrie, la loi de Gauss permet de déterminer une expression pour le champ électrique.
- Lire les exemples des pp. 762 à 767.
- Autres exemples :

22.18. **IDENTIFY:** According to Exercise 21.32, the Earth's electric field points towards its center. Since Mars's electric field is similar to that of Earth, we assume it points toward the center of Mars. Therefore the charge on Mars must be negative. We use Gauss's law to relate the electric flux to the charge causing it.

SET UP: Gauss's law is $\Phi_E = \dfrac{q}{\epsilon_0}$ and the electric flux is $\Phi_E = EA$.

EXECUTE: **(a)** Solving Gauss's law for q, putting in the numbers, and recalling that q is negative, gives $q = -\epsilon_0\Phi_E = -(3.63\times10^{16}\ \text{N}\cdot\text{m}^2/\text{C})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = -3.21\times10^5\ \text{C}$.

(b) Use the definition of electric flux to find the electric field. The area to use is the surface area of Mars. $E = \dfrac{\Phi_E}{A} = \dfrac{3.63\times10^{16}\ \text{N}\cdot\text{m}^2/\text{C}}{4\pi(3.40\times10^6\ \text{m})^2} = 2.50\times10^2\ \text{N/C}$

(c) The surface charge density on Mars is therefore

$\sigma = \dfrac{q}{A_{\text{Mars}}} = \dfrac{-3.21\times10^5\ \text{C}}{4\pi(3.40\times10^6\ \text{m})^2} = -2.21\times10^{-9}\ \text{C/m}^2$

EVALUATE: Even though the charge on Mars is very large, it is spread over a large area, giving a small surface charge density.

22.23. **IDENTIFY:** The electric field inside the conductor is zero, and all of its initial charge lies on its outer surface. The introduction of charge into the cavity induces charge onto the surface of the cavity, which induces an equal but opposite charge on the outer surface of the conductor. The net charge on the outer surface of the conductor is the sum of the positive charge initially there and the additional negative charge due to the introduction of the negative charge into the cavity.

(a) SET UP: First find the initial positive charge on the outer surface of the conductor using $q_i = \sigma A$, where A is the area of its outer surface. Then find the net charge on the surface after the negative charge has been introduced into the cavity. Finally use the definition of surface charge density.
EXECUTE: The original positive charge on the outer surface is

$$\underline{\underline{q_i = \sigma A = \sigma(4\pi r^2) = (6.37\times10^{-6}\ \text{C/m}^2)4\pi(0.250\ \text{m}^2) = 5.00\times10^{-6}\ \text{C/m}^2}}$$

After the introduction of $-0.500\ \mu\text{C}$ into the cavity, the outer charge is now

$$5.00\ \mu\text{C} - 0.500\ \mu\text{C} = 4.50\ \mu\text{C}$$

The surface charge density is now $\sigma = \dfrac{q}{A} = \dfrac{q}{4\pi r^2} = \dfrac{4.50\times10^{-6}\ \text{C}}{4\pi(0.250\ \text{m})^2} = 5.73\times10^{-6}\ \text{C/m}^2$

(b) SET UP: Using Gauss's law, the electric field is $E = \dfrac{\Phi_E}{A} = \dfrac{q}{\epsilon_0 A} = \dfrac{q}{\epsilon_0 4\pi r^2}$

EXECUTE: Substituting numbers gives

$$E = \frac{4.50\times10^{-6}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi)(0.250\ \text{m})^2} = 6.47\times10^5\ \text{N/C}.$$

(c) SET UP: We use Gauss's law again to find the flux. $\Phi_E = \dfrac{q}{\epsilon_0}$.

EXECUTE: Substituting numbers gives

$$\Phi_E = \frac{-0.500 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -5.65 \times 10^4\ \text{N}\cdot\text{m}^2/\text{C}^2.$$

EVALUATE: The excess charge on the conductor is still $+5.00\ \mu\text{C}$, as it originally was. The introduction of the $-0.500\ \mu\text{C}$ inside the cavity merely induced equal but opposite charges (for a net of zero) on the surfaces of the conductor.

22.24. **IDENTIFY:** We apply Gauss's law, taking the Gaussian surface beyond the cavity but inside the solid.

SET UP: Because of the symmetry of the charge, Gauss's law gives us $E = \dfrac{q_{\text{total}}}{\epsilon_0 A}$, where A is the surface area of a sphere of radius $R = 9.50$ cm centered on the point-charge, and q_{total} is the total charge contained within that sphere. This charge is the sum of the $-2.00\ \mu\text{C}$ point charge at the center of the cavity plus the charge within the solid between $r = 6.50$ cm and $R = 9.50$ cm. The charge within the solid is $q_{\text{solid}} = \rho V = \rho([4/3]\pi R^3 - [4/3]\pi r^3) = ([4\pi/3]\,\rho)(R^3 - r^3)$

EXECUTE: First find the charge within the solid between $r = 6.50$ cm and $R = 9.50$ cm:

$$q_{\text{solid}} = \frac{4\pi}{3}(7.35 \times 10^{-4}\ \text{C/m}^3)\left[(0.0950\ \text{m})^3 - (0.0650\ \text{m})^3\right] = 1.794 \times 10^{-6}\ \text{C},$$

Now find the total charge within the Gaussian surface:

$$q_{\text{total}} = q_{\text{solid}} + q_{\text{point}} = -2.00\ \mu\text{C} + 1.794\ \mu\text{C} = -0.2059\ \mu\text{C}$$

Now find the magnitude of the electric field from Gauss's law:

$$E = \frac{q}{\epsilon_0 A} = \frac{q}{\epsilon_0 4\pi r^2} = \frac{0.2059 \times 10^{-6}\ \text{C}}{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi)(0.0950\ \text{m})^2} = 2.05 \times 10^5\ \text{N/C}.$$

The fact that the charge is negative means that the electric field points radially inward.

EVALUATE: Because of the uniformity of the charge distribution, the charge beyond 9.50 cm does not contribute to the electric field.

22.28. **IDENTIFY:** Close to a finite sheet the field is the same as for an infinite sheet. Very far from a finite sheet the field is that of a point charge.

SET UP: For an infinite sheet, $E = \dfrac{\sigma}{2\epsilon_0}$. For a point charge, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$.

EXECUTE: **(a)** At a distance of 0.1 mm from the center, the sheet appears "infinite," so

$$E \approx \frac{q}{2\epsilon_0 A} = \frac{7.50 \times 10^{-9}\ \text{C}}{2\epsilon_0 (0.800\ \text{m})^2} = 662\ \text{N/C}.$$

(b) At a distance of 100 m from the center, the sheet looks like a point, so:

$$E \approx \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(7.50 \times 10^{-9}\ \text{C})}{(100\ \text{m})^2} = 6.75 \times 10^{-3}\ \text{N/C}.$$

(c) There would be no difference if the sheet was a conductor. The charge would automatically spread out evenly over both faces, giving it half the charge density on either face as the insulator but the same electric field. Far away, they both look like points with the same charge.

EVALUATE: The sheet can be treated as infinite at points where the distance to the sheet is much less than the distance to the edge of the sheet. The sheet can be treated as a point charge at points for which the distance to the sheet is much greater than the dimensions of the sheet.

22.5. Charges sur des conducteurs

- La section *Testing Gauss's Law Experimentally* peut être lue plus rapidement. Mettre l'accent sur la section *Field at the Surface of a Conductor*, et lire les exemple de la p. 771.
- Autres exemples :

22.29. **IDENTIFY:** Apply Gauss's law to a Gaussian surface and calculate E.

(a) SET UP: Consider the charge on a length l of the cylinder. This can be expressed as $q = \lambda l$. But since the surface area is $2\pi Rl$ it can also be expressed as $q = \sigma 2\pi Rl$. These two expressions must be equal, so $\lambda l = \sigma 2\pi Rl$ and $\lambda = 2\pi R\sigma$.

(b) Apply Gauss's law to a Gaussian surface that is a cylinder of length l, radius r, and whose axis coincides with the axis of the charge distribution, as shown in Figure 22.29.

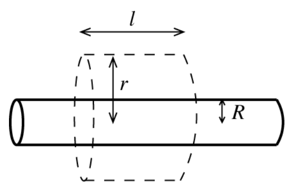

Figure 22.29

EXECUTE:

$Q_{\text{encl}} = \sigma(2\pi Rl)$

$\Phi_E = 2\pi rlE$

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2\pi rlE = \dfrac{\sigma(2\pi Rl)}{\epsilon_0}$

$$E = \frac{\sigma R}{\epsilon_0 r}$$

(c) EVALUATE: Example 22.6 shows that the electric field of an infinite line of charge is $E = \lambda / 2\pi\epsilon_0 r$. $\sigma = \dfrac{\lambda}{2\pi R}$, so $E = \dfrac{\sigma R}{\epsilon_0 r} = \dfrac{R}{\epsilon_0 r}\left(\dfrac{\lambda}{2\pi R}\right) = \dfrac{\lambda}{2\pi\epsilon_0 r}$, the same as for an infinite line of charge that is along the axis of the cylinder.

22.31. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: $E = 0$ in a conducting material.

EXECUTE: **(a)** Gauss's law says $+Q$ on inner surface, so $E = 0$ inside metal.

(b) The outside surface of the sphere is grounded, so no excess charge.

(c) Consider a Gaussian sphere with the $-Q$ charge at its center and radius less than the inner radius of the metal. This sphere encloses net charge $-Q$ so there is an electric field flux through it; there is electric field in the cavity.

(d) In an electrostatic situation $E = 0$ inside a conductor. A Gaussian sphere with the $-Q$ charge at its center and radius greater than the outer radius of the metal encloses zero net charge (the $-Q$ charge and the $+Q$ on the inner surface of the metal) so there is no flux through it and $E = 0$ outside the metal.

(e) No, $E = 0$ there. Yes, the charge has been shielded by the grounded conductor. There is nothing like positive and negative mass (the gravity force is always attractive), so this cannot be done for gravity.

EVALUATE: Field lines within the cavity terminate on the charges induced on the inner surface.