Chapitre 23 – Potentiel électrique [18 au 23 mai]

DEVOIR : 23.10, 23.16, 23.54, 23.61, 23.50

Le *potentiel électrique* est le travail par unité de charge (en J/C, ou volt). Ce concept est donc utile dans les problèmes de conservation d'énergie. Il est aussi très utile dans les circuits électriques, avec les lois de Kirchhoff, etc.). Notation: énergie cinétique *K*, énergie potentielle *U*, potentiel *V*.

23.1. Énergie potentielle électrique

- De façon générale, si une charge test q_0 se déplace du point *a* au point *b* dans un champ électrique **E**, alors la *différence d'énergie potentielle* (due à **E**) entre ces deux points est

$$U_b - U_a = -\Delta W_{a\to b} = -\int_a^b q_0 \vec{E}\cdot d\vec{l}$$

- Au besoin, réviser les concepts de travail et énergie potentielle.
- Si on connait la force, alors une intégrale (ci-dessus) permet d'obtenir l'énergie potentielle. Par conséquent, si on connait l'énergie potentielle, une dérivée permettra d'obtenir la force: l'Eq. (7.18) qui fait intervenir le *gradient*, qui reviendra à la section 23.5.
- Lire la section. Attention : ne pas oublier que l'Eq. (23.9) n'est valide que pour des charges ponctuelles! Si la source du champ n'est pas ponctuelle, alors il faut utiliser la formule ci-dessus.
- Exemples:

23.4. **IDENTIFY:** The work required is the change in electrical potential energy. The protons gain speed after being released because their potential energy is converted into kinetic energy.

(a) SET UP: Using the potential energy of a pair of point charges relative to infinity,

$U = (1/4\pi\epsilon_0)(qq_0/r)$. we have $W = \Delta U = U_2 - U_1 = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{e^2}{r_2} - \dfrac{e^2}{r_1}\right).$

EXECUTE: Factoring out the e^2 and substituting numbers gives

$$W = \left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2\left(\frac{1}{3.00\times10^{-15}\ \text{m}} - \frac{1}{2.00\times10^{-15}\ \text{m}}\right) = 7.68\times10^{-14}\ \text{J}$$

(b) SET UP: The protons have equal momentum, and since they have equal masses, they will have equal speeds and hence equal kinetic energy. $\Delta U = K_1 + K_2 = 2K = 2\left(\dfrac{1}{2}mv^2\right) = mv^2.$

EXECUTE: Solving for *v* gives $v = \sqrt{\dfrac{\Delta U}{m}} = \sqrt{\dfrac{7.68\times10^{-14}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}} = 6.78\times10^6$ m/s

EVALUATE: The potential energy may seem small (compared to macroscopic energies), but it is enough to give each proton a speed of nearly 7 million m/s.

23.11. **IDENTIFY:** Apply Eq.(23.2). The net work to bring the charges in from infinity is equal to the change in potential energy. The total potential energy is the sum of the potential energies of each pair of charges, calculated from Eq.(23.9).

SET UP: Let 1 be where all the charges are infinitely far apart. Let 2 be where the charges are at the corners of the triangle, as shown in Figure 23.11.

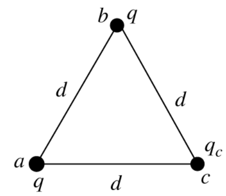

Figure 23.11

Let q_c be the third, unknown charge.

EXECUTE: $W = -\Delta U = -(U_2 - U_1)$

$$U_1 = 0$$

$$U_2 = U_{ab} + U_{ac} + U_{bc} = \frac{1}{4\pi\epsilon_0 d}(q^2 + 2qq_c)$$

Want $W = 0$, so $W = -(U_2 - U_1)$ gives $0 = -U_2$

$$0 = \frac{1}{4\pi\epsilon_0 d}(q^2 + 2qq_c)$$

$q^2 + 2qq_c = 0$ and $q_c = -q/2$.

EVALUATE: The potential energy for the two charges q is positive and for each q with q_c it is negative. There are two of the q, q_c terms so must have $q_c < q$.

23.2. Potentiel électrique

- Le potentiel électrique est l'énergie potentielle par unité de charge. Autrement dit, l'énergie potentielle U d'une charge test en un point est obtenue en multipliant la charge par la valeur du potentiel V en ce point. En ce sens, le V est à U ce que **E** est à $\mathbf{F_E}$.
- L'Eq. (23.17) est la plus générale. Attention: Les Eqs. (23.14) à (23.15) ne sont valides que pour des charges sources *ponctuelles*.
- Lire les exemples aux pp. 791 à 794.
- Autres exemples :

23.14. **IDENTIFY:** $\frac{W_{a\to b}}{q_0} = V_a - V_b$. For a point charge, $V = \frac{kq}{r}$.

SET UP: Each vacant corner is the same distance, 0.200 m, from each point charge.

EXECUTE: Taking the origin at the center of the square, the symmetry means that the potential is the same at the two corners not occupied by the $+5.00\ \mu\text{C}$ charges. This means that no net work is done is moving from one corner to the other.

EVALUATE: If the charge q_0 moves along a diagonal of the square, the electrical force does positive work for part of the path and negative work for another part of the path, but the net work done is zero.

23.21. **IDENTIFY:** $V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

SET UP: The locations of the changes and points A and B are sketched in Figure 23.21.

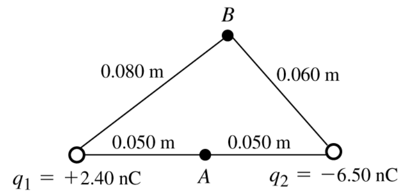

Figure 23.21

EXECUTE: **(a)** $V_A = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{A1}} + \frac{q_2}{r_{A2}}\right)$

$$V_A = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times 10^{-9}\ \text{C}}{0.050\ \text{m}} + \frac{-6.50\times 10^{-9}\ \text{C}}{0.050\ \text{m}}\right) = -737\ \text{V}$$

(b) $V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{B1}} + \frac{q_2}{r_{B2}}\right)$

$$V_B = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times 10^{-9}\ \text{C}}{0.080\ \text{m}} + \frac{-6.50\times 10^{-9}\ \text{C}}{0.060\ \text{m}}\right) = -704\ \text{V}$$

(c) IDENTIFY and **SET UP:** Use Eq.(23.13) and the results of parts (a) and (b) to calculate W.

EXECUTE: $W_{B\to A} = q'(V_B - V_A) = (2.50\times 10^{-9}\ \text{C})(-704\ \text{V} - (-737\ \text{V})) = +8.2\times 10^{-8}\ \text{J}$

EVALUATE: The electric force does positive work on the positive charge when it moves from higher potential (point B) to lower potential (point A).

23.23. **IDENTIFY:** For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: **(a)** The positions of the two charges are shown in Figure 23.23.

EXECUTE: **(b)** $V = \frac{kq}{r} + \frac{k(-q)}{r} = 0.$

(c) The potential along the x-axis is always zero, so a graph would be flat.

(d) If the two charges are interchanged, then the results of (b) and (c) still hold. The potential is zero.

EVALUATE: The potential is zero at any point on the x-axis because any point on the x-axis is equidistant from the two charges.

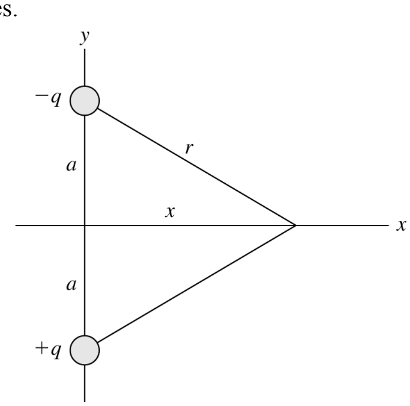

Figure 23.23

23.24. **IDENTIFY:** For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: Consider the distances from the point on the y-axis to each charge for the three regions $-a \le y \le a$ (between the two charges), $y > a$ (above both charges) and $y < -a$ (below both charges).

EXECUTE: **(a)** $|y| < a: V = \frac{kq}{(a+y)} - \frac{kq}{(a-y)} = \frac{2kqy}{y^2 - a^2}$. $y > a: V = \frac{kq}{(a+y)} - \frac{kq}{y-a} = \frac{-2kqa}{y^2 - a^2}$.

$y < -a: V = \frac{-kq}{(a+y)} - \frac{kq}{(-y+a)} = \frac{2kqa}{y^2 - a^2}$.

A general expression valid for any y is $V = k\left(\frac{-q}{|y-a|} + \frac{q}{|y+a|}\right)$.

(b) The graph of V versus y is sketched in Figure 23.24.

(c) $y >> a: V = \frac{-2kqa}{y^2 - a^2} \approx \frac{-2kqa}{y^2}$.

(d) If the charges are interchanged, then the potential is of the opposite sign.

EVALUATE: $V = 0$ at $y = 0$. $V \to +\infty$ as the positive charge is approached and $V \to -\infty$ as the negative charge is approached.

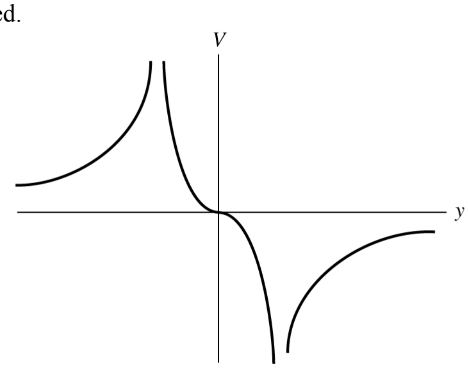

Figure 23.24

23.27. **IDENTIFY:** $K_a + qV_a = K_b + qV_b$.

SET UP: Let point a be at the cathode and let point b be at the anode. $K_a = 0$. $V_b - V_a = 295\text{ V}$. An electron has $q = -e$ and $m = 9.11 \times 10^{-31}\text{ kg}$.

EXECUTE: $K_b = q(V_a - V_b) = -(1.60 \times 10^{-19}\text{ C})(-295\text{ V}) = 4.72 \times 10^{-17}\text{ J}$. $K_b = \frac{1}{2}mv_b^2$, so

$v_b = \sqrt{\frac{2(4.72 \times 10^{-17}\text{ J})}{9.11 \times 10^{-31}\text{ kg}}} = 1.01 \times 10^7\text{ m/s}.$

EVALUATE: The negatively charged electron gains kinetic energy when it moves to higher potential.

23.29. **(a) IDENTIFY and SET UP:** The direction of $\vec{E}$ is always from high potential to low potential so point b is at higher potential.

(b) Apply Eq.(23.17) to relate $V_b - V_a$ to E.

EXECUTE: $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E\,dx = E(x_b - x_a).$

$$E = \frac{V_b - V_a}{x_b - x_a} = \frac{+240\text{ V}}{0.90\text{ m} - 0.60\text{ m}} = 800\text{ V/m}$$

(c) $W_{b\to a} = q(V_b - V_a) = (-0.200 \times 10^{-6}\text{ C})(+240\text{ V}) = -4.80 \times 10^{-5}\text{ J}.$

EVALUATE: The electric force does negative work on a negative charge when the negative charge moves from high potential (point *b*) to low potential (point *a*).

23.31. **IDENTIFY** and **SET UP:** Apply conservation of energy, Eq.(23.3). Use Eq.(23.12) to express *U* in terms of *V*.

(a) EXECUTE: $K_1 + qV_1 = K_2 + qV_2$

$q(V_1 - V_2) = K_2 - K_1;$ $\quad q = -1.602 \times 10^{-19}\ \text{C}$

$K_1 = \frac{1}{2} m_e v_1^2 = 4.099 \times 10^{-18}\ \text{J};$ $\quad K_2 = \frac{1}{2} m_e v_2^2 = 2.915 \times 10^{-17}\ \text{J}$

$$V_1 - V_2 = \frac{K_2 - K_1}{q} = -156\ \text{V}$$

EVALUATE: The electron gains kinetic energy when it moves to higher potential.

(b) EXECUTE: Now $K_1 = 2.915 \times 10^{-17}\ \text{J}, K_2 = 0$

$$V_1 - V_2 = \frac{K_2 - K_1}{q} = +182\ \text{V}$$

EVALUATE: The electron loses kinetic energy when it moves to lower potential.

23.3. Calculs de potentiels

- Section pratique. Lire l'encadré du bas de la p. 794 et l'exemple 23.8.
- Lire les exemples de pp. 796 à 798.
- Autres exemples :

23.34. **IDENTIFY:** Example 23.10 shows that for a line of charge, $V_a - V_b = \frac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$. Apply conservation of energy to the motion of the proton.

SET UP: Let point *a* be 18.0 cm from the line and let point *b* be at the distance of closest approach, where $K_b = 0$.

EXECUTE: **(a)** $K_a = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27}\ \text{kg})(1.50 \times 10^3\ \text{m/s})^2 = 1.88 \times 10^{-21}\ \text{J}$.

(b) $K_a + qV_a = K_b + qV_b$. $V_a - V_b = \frac{K_b - K_a}{q} = \frac{-1.88 \times 10^{-21}\ \text{J}}{1.60 \times 10^{-19}\ \text{C}} = -0.01175\ \text{V}$.

$$\ln(r_b / r_a) = \left(\frac{2\pi\epsilon_0}{\lambda}\right)(-0.01175\ \text{V}).$$

$$r_b = r_a \exp\left(\frac{2\pi\epsilon_0(-0.01175\ \text{V})}{\lambda}\right) = (0.180\ \text{m})\exp\left(-\frac{2\pi\epsilon_0(0.01175\ \text{V})}{5.00 \times 10^{-12}\ \text{C/m}}\right) = 0.158\ \text{m}.$$

EVALUATE: The potential increases with decreasing distance from the line of charge. As the positively charged proton approaches the line of charge it gains electrical potential energy and loses kinetic energy.

23.37. **IDENTIFY:** For points outside the cylinder, its electric field behaves like that of a line of charge. Since a voltmeter reads potential difference, that is what we need to calculate.

SET UP: The potential difference is $\Delta V = \frac{\lambda}{2\pi\epsilon_0}\ln\left(r_b / r_a\right)$.

EXECUTE: **(a)** Substituting numbers gives

$$\Delta V = \frac{\lambda}{2\pi\epsilon_0}\ln\left(r_b / r_a\right) = \left(8.50 \times 10^{-6}\ \text{C/m}\right)\left(2 \times 9.00 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\ln\left(\frac{10.0\ \text{cm}}{6.00\ \text{cm}}\right)$$

$$\Delta V = 7.82 \times 10^4 \text{ V} = 78{,}200 \text{ V} = 78.2 \text{ kV}$$

(b) $E = 0$ inside the cylinder, so the potential is constant there, meaning that the voltmeter reads zero.
EVALUATE: Caution! The fact that the voltmeter reads zero in part (b) does not mean that $V = 0$ inside the cylinder. The electric field is zero, but the potential is constant and equal to the potential at the surface.

23.41. **IDENTIFY** and **SET UP:** Use the result of Example 23.9 to relate the electric field between the plates to the potential difference between them and their separation. The force this field exerts on the particle is given by Eq.(21.3). Use the equation that precedes Eq.(23.17) to calculate the work.

EXECUTE: **(a)** From Example 23.9, $E = \dfrac{V_{ab}}{d} = \dfrac{360 \text{ V}}{0.0450 \text{ m}} = 8000 \text{ V/m}$

(b) $F = |q|E = (2.40 \times 10^{-9} \text{ C})(8000 \text{ V/m}) = +1.92 \times 10^{-5} \text{ N}$

(c) The electric field between the plates is shown in Figure 23.41.

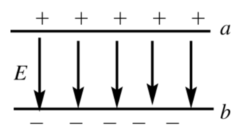

Figure 23.41

The plate with positive charge (plate *a*) is at higher potential. The electric field is directed from high potential toward low potential (or, $\vec{E}$ is from + charge toward − charge), so $\vec{E}$ points from *a* to *b*. Hence the force that $\vec{E}$ exerts on the positive charge is from *a* to *b*, so it does positive work.

$W = \int_a^b \vec{F} \cdot d\vec{l} = Fd$, where d is the separation between the plates.

$W = Fd = (1.92 \times 10^{-5} \text{ N})(0.0450 \text{ m}) = +8.64 \times 10^{-7} \text{ J}$

(d) $V_a - V_b = +360 \text{ V}$ (plate *a* is at higher potential)

$\Delta U = U_b - U_a = q(V_b - V_a) = (2.40 \times 10^{-9} \text{ C})(-360 \text{ V}) = -8.64 \times 10^{-7} \text{ J}.$

EVALUATE: We see that $W_{a \to b} = -(U_b - U_a) = U_a - U_b$.

23.44. **IDENTIFY:** Example 23.8 shows that the potential of a solid conducting sphere is the same at every point inside the sphere and is equal to its value $V = q / 2\pi\epsilon_0 R$ at the surface. Use the given value of E to find q.
SET UP: For negative charge the electric field is directed toward the charge.
For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

EXECUTE: $E = \dfrac{q}{4\pi\epsilon_0 r^2}$ and $q = 4\pi\epsilon_0 E r^2 = \dfrac{(3800 \text{ N/C})(0.200 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 1.69 \times 10^{-8} \text{ C}$.

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking ∞ as zero, is $V = \dfrac{q}{4\pi\epsilon_0 r} = \dfrac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-1.69 \times 10^{-8} \text{ C})}{0.200 \text{ m}} = -760 \text{ V}$ at the surface of the sphere. Since the charge all resides on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be -760 V.
EVALUATE: Inside the sphere the electric field is zero and the potential is constant.

23.4. Surfaces équipotentielles

- L'analogie des courbes de contour (Fig. 23.23) aide à avoir une compréhension intuitive. Par exemples, des courbes rapprochées correspondent à une pente plus raide; dans le contexte électrique, celà signifie **E** plus élevé.

- Lire rapidement la section, qui est plutôt de nature conceptuelle.

23.5. Champ électrique comme gradient du potentiel

- Si *V* est connu, alors Eq. (23.20) donne **E**. Si *V* (et **E**) est radial, alors on utilise Eq. (23.23).
- Lire les deux exemples.
- Autres exemples:

23.47 **IDENTIFY** and **SET UP:** Use Eq.(23.19) to calculate the components of $\vec{E}$.

EXECUTE: $V = Axy - Bx^2 + Cy$

(a) $E_x = -\dfrac{\partial V}{\partial x} = -Ay + 2Bx$

$Ey = -\dfrac{\partial V}{\partial y} = -Ax - C$

$E_z = -\dfrac{\partial V}{\partial z} = 0$

(b) $E = 0$ requires that $E_x = E_y = E_z = 0$.

$E_z = 0$ everywhere.

$E_y = 0$ at $x = -C/A$.

And E_x is also equal zero for this *x*, any value of *z*, and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2$.

EVALUATE: *V* doesn't depend on *z* so $E_z = 0$ everywhere.

23.48. **IDENTIFY:** Apply Eq.(21.19).

SET UP: Eq.(21.7) says $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}\hat{r}$ is the electric field due to a point charge *q*.

EXECUTE: **(a)** $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial}{\partial x}\left(\dfrac{kQ}{\sqrt{x^2 + y^2 + z^2}}\right) = \dfrac{kQx}{(x^2 + y^2 + z^2)^{3/2}} = \dfrac{kQx}{r^3}$.

Similarly, $E_y = \dfrac{kQy}{r^3}$ and $E_z = \dfrac{kQz}{r^3}$.

(b) From part (a), $E = \dfrac{kQ}{r^2}\left(\dfrac{x\hat{i}}{r} + \dfrac{y\hat{j}}{r} + \dfrac{z\hat{k}}{r}\right) = \dfrac{kQ}{r^2}\hat{r}$, which agrees with Equation (21.7).

EVALUATE: *V* is a scalar. $\vec{E}$ is a vector and has components.

23.49. **IDENTIFY** and **SET UP:** For a solid metal sphere or for a spherical shell, $V = \dfrac{kq}{r}$ outside the sphere and $V = \dfrac{kq}{R}$ at all points inside the sphere, where *R* is the radius of the sphere. When the electric field is radial, $E = -\dfrac{\partial V}{\partial r}$.

EXECUTE: **(a)** (i) $r < r_a$: This region is inside both spheres. $V = \dfrac{kq}{r_a} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$.

(ii) $r_a < r < r_b$: This region is outside the inner shell and inside the outer shell.

$V = \dfrac{kq}{r} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$. (iii) $r > r_b$: This region is outside both spheres and $V = 0$ since outside a sphere the potential is the same as for point charge. Therefore the potential is the same as for two oppositely charged point charges at the same location. These potentials cancel.

(b) $V_a = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$ and $V_b = 0$, so $V_{ab} = \frac{1}{4\pi\epsilon_0} q\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$.

(c) Between the spheres $r_a < r < r_b$ and $V = kq\left(\frac{1}{r} - \frac{1}{r_b}\right)$.

$$E = -\frac{\partial V}{\partial r} = -\frac{q}{4\pi\epsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{r_b}\right) = +\frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{V_{ab}}{\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}\frac{1}{r^2}.$$

(d) From Equation (23.23): $E = 0$, since V is constant (zero) outside the spheres.

(e) If the outer charge is different, then outside the outer sphere the potential is no longer zero but is $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} - \frac{1}{4\pi\epsilon_0}\frac{Q}{r} = \frac{1}{4\pi\epsilon_0}\frac{(q-Q)}{r}$. All potentials inside the outer shell are just shifted by an amount $V = -\frac{1}{4\pi\epsilon_0}\frac{Q}{r_b}$. Therefore relative potentials within the shells are not affected. Thus (b) and (c) do not change. However, now that the potential does vary outside the spheres, there is an electric field there:

$$E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{kq}{r} + \frac{-kQ}{r}\right) = \frac{kq}{r^2}\left(1 - \frac{Q}{q}\right) = \frac{k}{r^2}(q - Q).$$

EVALUATE: In part (a) the potential is greater than zero for all $r < r_b$.