

## Chapitre 27 – Champs et forces magnétiques [2 au 7 juin]

DEVOIR : 27.53; 27.57; 27.59; 27.70; 27.50

### 27.1. Magnétisme

- Fig. 27.4: Il n'y a pas de monopôle magnétique, mais que des dipôles
- Fig. 27.1: Comme pour les dipôles électriques, les pôles opposés s'attirent et les pôles de mêmes signes se repoussent.
- Fig. 27.3: (1) Pôle nord (sud) géographique = pôle sud (nord) magnétique, (2) les lignes de champ magnétiques s'éloignent de N et pointent vers S.

### 27.2. Champs magnétiques

- Eq. (27.2) donne la force magnétique sur une charge. (Grandeur donnée par Eq. (27.1))
- Fig. 27.7 : direction de  $\mathbf{F}$  par la main droite. Attention si la charge est négative: il faut renverser la direction.
- Eq. (27.4) : charge dans un champ électrique et un champ magnétique.
- Lire l'exemple 27.1
- Autres exemples:

27.7. IDENTIFY: Apply  $\vec{F} = q\vec{v} \times \vec{B}$ .

SET UP:  $\vec{v} = v_y \hat{j}$ , with  $v_y = -3.80 \times 10^3$  m/s.  $F_x = +7.60 \times 10^{-3}$  N,  $F_y = 0$ , and

$$F_z = -5.20 \times 10^{-3} \text{ N}.$$

EXECUTE: (a)  $F_x = q(v_y B_z - v_z B_y) = qv_y B_z$ .

$$B_z = F_x / qv_y = (7.60 \times 10^{-3} \text{ N}) / [(7.80 \times 10^{-6} \text{ C})(-3.80 \times 10^3 \text{ m/s})] = -0.256 \text{ T}$$

$F_y = q(v_z B_x - v_x B_z) = 0$ , which is consistent with  $\vec{F}$  as given in the problem. There is no force component along the direction of the velocity.

$$F_z = q(v_x B_y - v_y B_x) = -qv_y B_x. B_x = -F_z / qv_y = -0.175 \text{ T}.$$

(b)  $B_y$  is not determined. No force due to this component of  $\vec{B}$  along  $\vec{v}$ ; measurement of the force tells us nothing about  $B_y$ .

(c)  $\vec{B} \cdot \vec{F} = B_x F_x + B_y F_y + B_z F_z = (-0.175 \text{ T})(+7.60 \times 10^{-3} \text{ N}) + (-0.256 \text{ T})(-5.20 \times 10^{-3} \text{ N})$

$$\vec{B} \cdot \vec{F} = 0. \vec{B} \text{ and } \vec{F} \text{ are perpendicular (angle is } 90^\circ).$$

EVALUATE: The force is perpendicular to both  $\vec{v}$  and  $\vec{B}$ , so  $\vec{v} \cdot \vec{F}$  is also zero.

27.8. IDENTIFY and SET UP:  $\vec{F} = q\vec{v} \times \vec{B} = qB_z[v_x(\hat{i} \times \hat{k}) + v_y(\hat{j} \times \hat{k}) + v_z(\hat{k} \times \hat{k})] = qB_z[v_x(-\hat{j}) + v_y(\hat{i})]$ .

EXECUTE: (a) Set the expression for  $\vec{F}$  equal to the given value of  $\vec{F}$  to obtain:

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40 \times 10^{-7} \text{ N})}{-(-5.60 \times 10^{-9} \text{ C})(-1.25 \text{ T})} = -106 \text{ m/s}$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40 \times 10^{-7} \text{ N})}{(-5.60 \times 10^{-9} \text{ C})(-1.25 \text{ T})} = -48.6 \text{ m/s}.$$

(b)  $v_z$  does not contribute to the force, so is not determined by a measurement of  $\vec{F}$ .

$$(c) \vec{v} \cdot \vec{F} = v_x F_x + v_y F_y + v_z F_z = \frac{F_y}{-qB_z} F_x + \frac{F_x}{qB_z} F_y = 0; \theta = 90^\circ.$$

EVALUATE: The force is perpendicular to both  $\vec{v}$  and  $\vec{B}$ , so  $\vec{B} \cdot \vec{F}$  is also zero.

### 27.3. Lignes de champ magnétique et flux magnétique

- Le flux magnétique n'est pas utile avec la loi de Gauss, mais plutôt avec la loi d'induction magnétique, que nous verrons au Chapitre 29.
- Fig. 27.12: Attention, les lignes pointent dans la direction de  $\mathbf{B}$ , pas de  $\mathbf{F}_B$
- Eq. (27.6) : flux magnétique (analogue au flux électrique Eq. (22.5))
- Eq. (27.8): loi de Gauss magnétique (car  $Q_{\text{mag}} = 0$ )
- Lire l'exemple 27.2
- Autres exemples:

27.10. IDENTIFY: Magnetic field lines are closed loops, so the net flux through any closed surface is zero.

SET UP: Let magnetic field directed out of the enclosed volume correspond to positive flux and magnetic field directed into the volume correspond to negative flux.

EXECUTE: (a) The total flux must be zero, so the flux through the remaining surfaces must be  $-0.120 \text{ Wb}$ .

(b) The shape of the surface is unimportant, just that it is closed.

(c) One possibility is sketched in Figure 27.10.

EVALUATE: In Figure 27.10 all the field lines that enter the cube also exit through the surface of the cube.

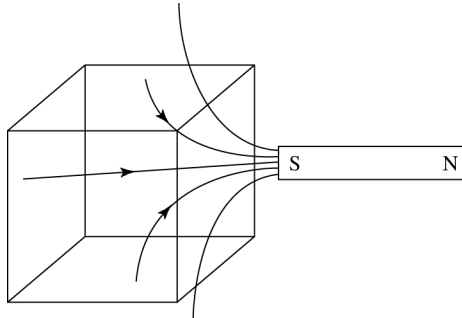


Figure 27.10

27.11. IDENTIFY and SET UP:  $\Phi_B = \int \vec{B} \cdot d\vec{A}$

Circular area in the  $xy$ -plane, so  $A = \pi r^2 = \pi(0.0650 \text{ m})^2 = 0.01327 \text{ m}^2$  and  $d\vec{A}$  is in the  $z$ -direction.

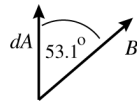
Use Eq.(1.18) to calculate the scalar product.

EXECUTE: (a)  $\vec{B} = (0.230 \text{ T})\hat{k}$ ;  $\vec{B}$  and  $d\vec{A}$  are parallel ( $\phi = 0^\circ$ ) so  $\vec{B} \cdot d\vec{A} = B dA$ .

$B$  is constant over the circular area so

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B dA = B \int dA = BA = (0.230 \text{ T})(0.01327 \text{ m}^2) = 3.05 \times 10^{-3} \text{ Wb}$$

(b) The directions of  $\vec{B}$  and  $d\vec{A}$  are shown in Figure 27.11a.



$$\vec{B} \cdot d\vec{A} = B \cos \phi dA$$

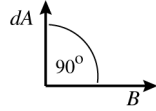
with  $\phi = 53.1^\circ$

Figure 27.11a

$B$  and  $\phi$  are constant over the circular area so  $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos \phi dA = B \cos \phi \int dA = B \cos \phi A$

$$\Phi_B = (0.230 \text{ T}) \cos 53.1^\circ (0.01327 \text{ m}^2) = 1.83 \times 10^{-3} \text{ Wb}$$

(c) The directions of  $\vec{B}$  and  $d\vec{A}$  are shown in Figure 27.11b.



$\vec{B} \cdot d\vec{A} = 0$  since  $d\vec{A}$  and  $\vec{B}$  are perpendicular ( $\phi = 90^\circ$ )

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = 0.$$

Figure 27.11b

**EVALUATE:** Magnetic flux is a measure of how many magnetic field lines pass through the surface. It is maximum when  $\vec{B}$  is perpendicular to the plane of the loop (part a) and is zero when  $\vec{B}$  is parallel to the plane of the loop (part c).

**27.12. IDENTIFY:** When  $\vec{B}$  is uniform across the surface,  $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \phi$ .

**SET UP:**  $\vec{A}$  is normal to the surface and is directed outward from the enclosed volume. For surface  $abcd$ ,  $\vec{A} = -A\hat{i}$ . For surface  $befc$ ,  $\vec{A} = -A\hat{k}$ . For surface  $aefd$ ,  $\cos \phi = 3/5$  and the flux is positive.

**EXECUTE:** (a)  $\Phi_B(abcd) = \vec{B} \cdot \vec{A} = 0$ .

(b)  $\Phi_B(befc) = \vec{B} \cdot \vec{A} = -(0.128 \text{ T})(0.300 \text{ m})(0.300 \text{ m}) = -0.0115 \text{ Wb}$ .

(c)  $\Phi_B(aefd) = \vec{B} \cdot \vec{A} = BA \cos \phi = \frac{3}{5}(0.128 \text{ T})(0.500 \text{ m})(0.300 \text{ m}) = +0.0115 \text{ Wb}$ .

(d) The net flux through the rest of the surfaces is zero since they are parallel to the  $x$ -axis. The total flux is the sum of all parts above, which is zero.

#### 27.4. Mouvement de particules chargées dans un champ magnétique

- Eq. (27.11) prouvée en utilisant la Fig. 27.17 et Eq. (27.10). (Une charge dans un champ magnétique tourne ce qui implique une accélération centripète.)
- Fig. 27.19 : mouvement d'une charge. Applications dans les Fig. 27.20 et 27.21.
- Exemples de la p. 928
- Autres exemples:

**27.15. (a) IDENTIFY:** Apply Eq.(27.2) to relate the magnetic force  $\vec{F}$  to the directions of  $\vec{v}$  and  $\vec{B}$ . The electron has negative charge so  $\vec{F}$  is opposite to the direction of  $\vec{v} \times \vec{B}$ . For motion in an arc of a circle the acceleration is toward the center of the arc so  $\vec{F}$  must be in this direction.  $a = v^2 / R$ .

**SET UP:**

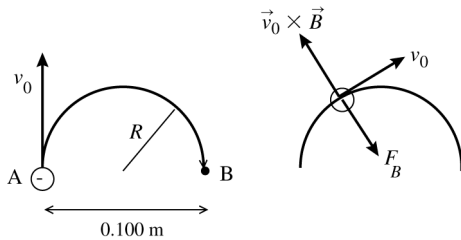


Figure 27.15

As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of  $\vec{v}_0 \times \vec{B}$  at a point along the path is shown in Figure 27.15.

**EXECUTE:** For circular motion the acceleration of the electron  $\vec{a}_{\text{rad}}$  is directed in toward the center of the circle. Thus the force  $\vec{F}_B$  exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since  $q$  is negative,  $\vec{F}_B$  is opposite to the direction given by the right-hand rule for  $\vec{v}_0 \times \vec{B}$ . Thus  $\vec{B}$  is directed into the page. Apply Newton's 2nd law to calculate the magnitude of

$$\vec{B} : \sum \vec{F} = m\vec{a} \text{ gives } \sum F_{\text{rad}} = ma$$

$$F_B = m(v^2/R)$$

$$F_B = |q|vB\sin\phi = |q|vB, \text{ so } |q|vB = m(v^2/R)$$

$$B = \frac{mv}{|q|R} = \frac{(9.109 \times 10^{-31} \text{ kg})(1.41 \times 10^6 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(0.050 \text{ m})} = 1.60 \times 10^{-4} \text{ T}$$

**(b) IDENTIFY and SET UP:** The speed of the electron as it moves along the path is constant. ( $\vec{F}_B$  changes the direction of  $\vec{v}$  but not its magnitude.) The time is given by the distance divided by  $v_0$ .

**EXECUTE:** The distance along the semicircular path is  $\pi R$ , so

$$t = \frac{\pi R}{v_0} = \frac{\pi(0.050 \text{ m})}{1.41 \times 10^6 \text{ m/s}} = 1.11 \times 10^{-7} \text{ s}$$

**EVALUATE:** The magnetic field required increases when  $v$  increases or  $R$  decreases and also depends on the mass to charge ratio of the particle.

**27.20. IDENTIFY:**  $F = |q|vB\sin\phi$ . The direction of  $\vec{F}$  is given by the right-hand rule.

**SET UP:** An electron has  $q = -e$ .

**EXECUTE: (a)**  $F = |q|vB\sin\phi$ .  $B = \frac{F}{|q|v\sin\phi} = \frac{0.00320 \times 10^{-9} \text{ N}}{8(1.60 \times 10^{-19} \text{ C})(500,000 \text{ m/s})\sin 90^\circ} = 5.00 \text{ T}$ . If the angle  $\phi$  is less than  $90^\circ$ , a larger field is needed to produce the same force. The direction of the field must be toward the south so that  $\vec{v} \times \vec{B}$  is downward.

**(b)**  $F = |q|vB\sin\phi$ .  $v = \frac{F}{|q|B\sin\phi} = \frac{4.60 \times 10^{-12} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(2.10 \text{ T})\sin 90^\circ} = 1.37 \times 10^7 \text{ m/s}$ . If  $\phi$  is less than

$90^\circ$ , the speed would have to be larger to have the same force. The force is upward, so  $\vec{v} \times \vec{B}$  must be downward since the electron is negative, and the velocity must be toward the south.

**EVALUATE:** The component of  $\vec{B}$  along the direction of  $\vec{v}$  produces no force and the component of  $\vec{v}$  along the direction of  $\vec{B}$  produces no force.

**27.22. IDENTIFY:** For motion in an arc of a circle,  $a = \frac{v^2}{R}$  and the net force is radially inward, toward the center of the circle.

**SET UP:** The direction of the force is shown in Figure 27.22. The mass of a proton is  $1.67 \times 10^{-27} \text{ kg}$ .

**EXECUTE: (a)**  $\vec{F}$  is opposite to the right-hand rule direction, so the charge is negative.  $\vec{F} = m\vec{a}$  gives

$$|q|vB\sin\phi = m\frac{v^2}{R}. \phi = 90^\circ \text{ and } v = \frac{|q|BR}{m} = \frac{3(1.60 \times 10^{-19} \text{ C})(0.250 \text{ T})(0.475 \text{ m})}{12(1.67 \times 10^{-27} \text{ kg})} = 2.84 \times 10^6 \text{ m/s}.$$

**(b)**  $F_B = |q|vB\sin\phi = 3(1.60 \times 10^{-19} \text{ C})(2.84 \times 10^6 \text{ m/s})(0.250 \text{ T})\sin 90^\circ = 3.41 \times 10^{-13} \text{ N}$ .

$w = mg = 12(1.67 \times 10^{-27} \text{ kg})(9.80 \text{ m/s}^2) = 1.96 \times 10^{-25} \text{ N}$ . The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

**EVALUATE: (c)** The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

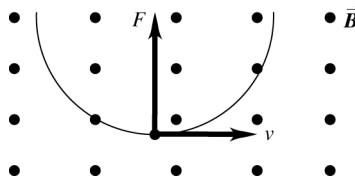


Figure 27.22

**27.25. IDENTIFY:** When a particle of charge  $-e$  is accelerated through a potential difference of magnitude  $V$ , it gains kinetic energy  $eV$ . When it moves in a circular path of radius  $R$ , its acceleration is  $\frac{v^2}{R}$ .

**SET UP:** An electron has charge  $q = -e = -1.60 \times 10^{-19}$  C and mass  $9.11 \times 10^{-31}$  kg.

**EXECUTE:**  $\frac{1}{2}mv^2 = eV$  and  $v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^3 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = 2.65 \times 10^7 \text{ m/s}$ .

$\vec{F} = m\vec{a}$  gives  $|q|vB \sin\phi = m\frac{v^2}{R}$ .  $\phi = 90^\circ$  and

$$B = \frac{mv}{|q|R} = \frac{(9.11 \times 10^{-31} \text{ kg})(2.65 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.180 \text{ m})} = 8.38 \times 10^{-4} \text{ T}.$$

**EVALUATE:** The smaller the radius of the circular path, the larger the magnitude of the magnetic field that is required.

**27.27. (a) IDENTIFY and SET UP:** Eq.(27.4) gives the total force on the proton. At  $t = 0$ ,

$$\vec{F} = q\vec{v} \times \vec{B} = q(v_x\hat{i} + v_z\hat{k}) \times B_x\hat{i} = qv_zB_x\hat{j}.$$

$$\vec{F} = (1.60 \times 10^{-19} \text{ C})(2.00 \times 10^5 \text{ m/s})(0.500 \text{ T})\hat{j} = (1.60 \times 10^{-14} \text{ N})\hat{j}.$$

**(b)** Yes. The electric field exerts a force in the direction of the electric field, since the charge of the proton is positive and there is a component of acceleration in this direction.

**(c) EXECUTE:** In the plane perpendicular to  $\vec{B}$  (the  $yz$ -plane) the motion is circular. But there is a velocity component in the direction of  $\vec{B}$ , so the motion is a helix. The electric field in the  $+\hat{i}$  direction exerts a force in the  $+\hat{i}$  direction. This force produces an acceleration in the  $+\hat{i}$  direction and this causes the pitch of the helix to vary. The force does not affect the circular motion in the  $yz$ -plane, so the electric field does not affect the radius of the helix.

**(d) IDENTIFY and SET UP:** Eq.(27.12) and  $T = 2\pi / \omega$  to calculate the period of the motion. Calculate  $a_x$  produced by the electric force and use a constant acceleration equation to calculate the displacement in the  $x$ -direction in time  $T/2$ .

**EXECUTE:** Calculate the period  $T$ :  $\omega = |q|B / m$

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} = \frac{2\pi(1.67 \times 10^{-27} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})} = 1.312 \times 10^{-7} \text{ s. Then } t = T / 2 = 6.56 \times 10^{-8} \text{ s.}$$

$$v_{0x} = 1.50 \times 10^5 \text{ m/s}$$

$$a_x = \frac{F_x}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^4 \text{ V/m})}{1.67 \times 10^{-27} \text{ kg}} = +1.916 \times 10^{12} \text{ m/s}^2$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = (1.50 \times 10^5 \text{ m/s})(6.56 \times 10^{-8} \text{ s}) + \frac{1}{2}(1.916 \times 10^{12} \text{ m/s}^2)(6.56 \times 10^{-8} \text{ s})^2 = 1.40 \text{ cm}$$

**EVALUATE:** The electric and magnetic fields are in the same direction but produce forces that are in perpendicular directions to each other.

## 27.5. Applications du mouvement des particules chargées

- Lire la section et les exemples
- Autres exemples:

**27.28. IDENTIFY:** For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

**SET UP:**  $v = E/B$  for no deflection. With only the magnetic force,  $|q|vB = mv^2/R$

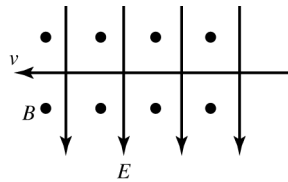
**EXECUTE:** (a)  $v = E/B = (1.56 \times 10^4 \text{ V/m}) / (4.62 \times 10^{-3} \text{ T}) = 3.38 \times 10^6 \text{ m/s}$ .

(b) The directions of the three vectors  $\vec{v}$ ,  $\vec{E}$  and  $\vec{B}$  are sketched in Figure 27.28.

$$(c) R = \frac{mv}{|q|B} = \frac{(9.11 \times 10^{-31} \text{ kg})(3.38 \times 10^6 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(4.62 \times 10^{-3} \text{ T})} = 4.17 \times 10^{-3} \text{ m}.$$

$$T = \frac{2\pi m}{|q|B} = \frac{2\pi R}{v} = \frac{2\pi(4.17 \times 10^{-3} \text{ m})}{(3.38 \times 10^6 \text{ m/s})} = 7.74 \times 10^{-9} \text{ s}.$$

**EVALUATE:** For the field directions shown in Figure 27.28, the electric force is toward the top of the page and the magnetic force is toward the bottom of the page.



**Figure 27.28**

**27.31. IDENTIFY and SET UP:** Use the fields in the velocity selector to find the speed  $v$  of the particles that pass through. Apply Newton's 2nd law with  $a = v^2/R$  to the circular motion in the second region of the spectrometer. Solve for the mass  $m$  of the ion.

**EXECUTE:** In the velocity selector  $|q|E = |q|vB$ .

$$v = \frac{E}{B} = \frac{1.12 \times 10^5 \text{ V/m}}{0.540 \text{ T}} = 2.074 \times 10^5 \text{ m/s}$$

In the region of the circular path  $\sum \vec{F} = m\vec{a}$  gives  $|q|vB = m(v^2/R)$  so  $m = |q|RB/v$

Singly charged ion, so  $|q| = +e = 1.602 \times 10^{-19} \text{ C}$

$$m = \frac{(1.602 \times 10^{-19} \text{ C})(0.310 \text{ m})(0.540 \text{ T})}{2.074 \times 10^5 \text{ m/s}} = 1.29 \times 10^{-25} \text{ kg}$$

Mass number = mass in atomic mass units, so is  $\frac{1.29 \times 10^{-25} \text{ kg}}{1.66 \times 10^{-27} \text{ kg}} = 78$ .

**EVALUATE:** Appendix D gives the average atomic mass of selenium to be 78.96. One of its isotopes has atomic mass 78.

## 27.6. La force magnétique sur un conducteur

- Eq. (27.19) : force sur un fil de longueur finie
- Eq. (27.20): force sur un fil de longueur infinitésimale
- Lire les exemple 27.7 et 27.8
- Autres exemples :

**27.36. IDENTIFY and SET UP:**  $F = I\ell B \sin \phi$ . The direction of  $\vec{F}$  is given by applying the right-hand rule to the directions of  $I$  and  $\vec{B}$ .

**EXECUTE:** (a) The current and field directions are shown in Figure 27.36a. The right-hand rule gives that  $\vec{F}$  is directed to the south, as shown.  $\phi = 90^\circ$  and

$$F = (1.20\text{A})(1.00 \times 10^{-2} \text{ m})(0.588 \text{ T}) = 7.06 \times 10^{-3} \text{ N}.$$

(b) The right-hand rule gives that  $\vec{F}$  is directed to the west, as shown in Figure 27.36b.  $\phi = 90^\circ$  and

$$F = 7.06 \times 10^{-3} \text{ N}, \text{ the same as in part (a).}$$

(c) The current and field directions are shown in Figure 27.36c. The right-hand rule gives that  $\vec{F}$  is  $60.0^\circ$  north of west.  $\phi = 90^\circ$  so  $F = 7.06 \times 10^{-3} \text{ N}$ , the same as in part (a).

**EVALUATE:** In each case the current direction is perpendicular to the magnetic field. The magnitude of the magnetic force is the same in each case but its direction depends on the direction of the magnetic field.

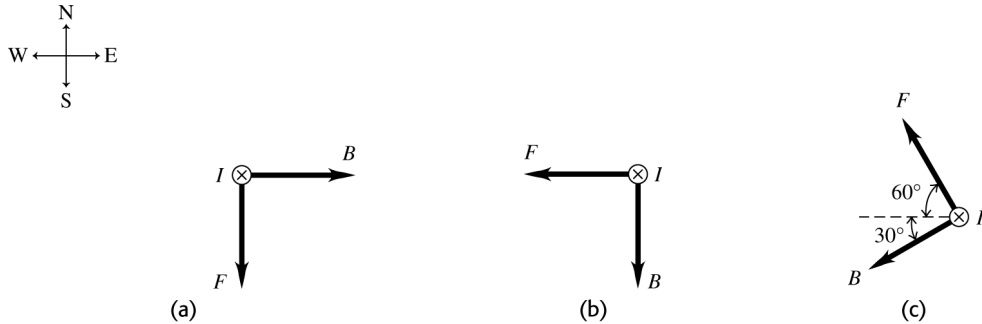


Figure 27.36

**27.40. IDENTIFY:** The magnetic force  $\vec{F}_B$  must be upward and equal to  $mg$ . The direction of  $\vec{F}_B$  is determined by the direction of  $I$  in the circuit.

**SET UP:**  $F_B = IlB \sin \phi$ , with  $\phi = 90^\circ$ .  $I = \frac{V}{R}$ , where  $V$  is the battery voltage.

**EXECUTE:** (a) The forces are shown in Figure 27.40. The current  $I$  in the bar must be to the right to produce  $\vec{F}_B$  upward. To produce current in this direction, point  $a$  must be the positive terminal of the battery.

$$(b) F_B = mg \Rightarrow IlB = mg \Rightarrow m = \frac{IlB}{g} = \frac{VlB}{Rg} = \frac{(175 \text{ V})(0.600 \text{ m})(1.50 \text{ T})}{(5.00 \Omega)(9.80 \text{ m/s}^2)} = 3.21 \text{ kg}.$$

**EVALUATE:** If the battery had opposite polarity, with point  $a$  as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

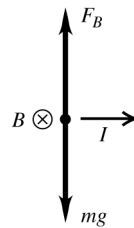


Figure 27.40

**27.67. IDENTIFY:** The force exerted by the magnetic field is given by Eq.(27.19). The net force on the wire must be zero.

**SET UP:** For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in Figure 27.61 in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.67a.

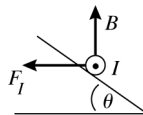


Figure 27.67a

The free-body diagram for the wire is given in Figure 27.67b.

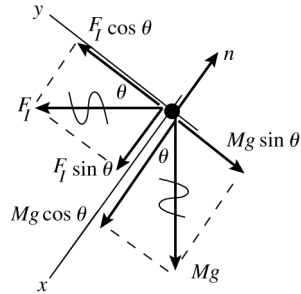


Figure 27.67b

EXECUTE:  $\sum F_y = 0$   
 $F_I \cos \theta - Mg \sin \theta = 0$   
 $F_I = ILB \sin \phi$   
 $\phi = 90^\circ$  since  $\vec{B}$  is perpendicular to the current direction.

Thus  $(ILB) \cos \theta - Mg \sin \theta = 0$  and  $I = \frac{Mg \tan \theta}{LB}$

EVALUATE: The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle  $\theta$  increases there is a larger component of  $Mg$  down the incline and the component of  $F_I$  up the incline is smaller;  $I$  must increase with  $\theta$  to compensate. As  $\theta \rightarrow 0, I \rightarrow 0$  and as  $\theta \rightarrow 90^\circ, I \rightarrow \infty$ .

27.7. Force et moment de force sur une boucle de courant

- Eq. (27.26) donne le moment de force g n r  par un champ  $\mathbf{B}$  sur une boucle de courant de moment magn tique  $\mu$ . Le moment magn tique est un vecteur de grandeur donn e par Eq. (27.24) et la direction illustr e   la Fig. 27.32
- Eq. (27.27) :  nergie potentielle d' un moment magn tique dans un champ  $\mathbf{B}$
- Lire la p. 938 et les exemple 27.9 et 27.10. Omettre le reste de la section.
- Autres exemples:

27.44. IDENTIFY:  $\tau = IAB \sin \phi$ , where  $\phi$  is the angle between  $\vec{B}$  and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.44a for its original position and in Figure 27.44b after it has rotated 30.0 .

EXECUTE: (a) The forces on each side of the coil are shown in Figure 27.44a.  $\vec{F}_1 + \vec{F}_2 = 0$  and  $\vec{F}_3 + \vec{F}_4 = 0$ . The net force on the coil is zero.  $\phi = 0^\circ$  and  $\sin \phi = 0$ , so  $\tau = 0$ . The forces on the coil produce no torque.

(b) The net force is still zero.  $\phi = 30.0^\circ$  and the net torque is  $\tau = (1)(1.40 \text{ A})(0.220 \text{ m})(0.350 \text{ m})(1.50 \text{ T})\sin 30.0^\circ = 0.0808 \text{ N} \cdot \text{m}$ . The net torque is clockwise in Figure 27.44b and is directed so as to increase the angle  $\phi$ .

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

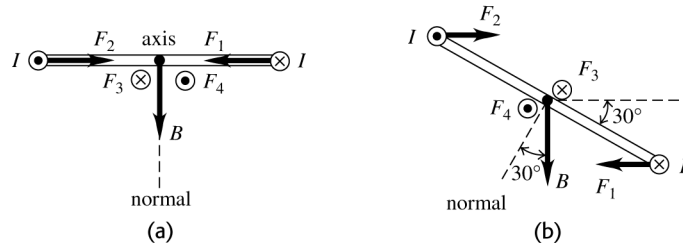


Figure 27.44



**27.45. IDENTIFY:** The magnetic field exerts a torque on the current-carrying coil, which causes it to turn. We can use the rotational form of Newton's second law to find the angular acceleration of the coil.

**SET UP:** The magnetic torque is given by  $\vec{\tau} = \vec{\mu} \times \vec{B}$ , and the rotational form of Newton's second law is  $\sum \tau = I\alpha$ . The magnetic field is parallel to the plane of the loop.

**EXECUTE:** (a) The coil rotates about axis  $A_2$  because the only torque is along top and bottom sides of the coil.

(b) To find the moment of inertia of the coil, treat the two 1.00-m segments as point-masses (since all the points in them are 0.250 m from the rotation axis) and the two 0.500-m segments as thin uniform bars rotated about their centers. Since the coil is uniform, the mass of each segment is proportional to its fraction of the total perimeter of the coil. Each 1.00-m segment is 1/3 of the total perimeter, so its mass is  $(1/3)(210 \text{ g}) = 70 \text{ g} = 0.070 \text{ kg}$ . The mass of each 0.500-m segment is half this amount, or 0.035 kg. The result is

$$I = 2(0.070 \text{ kg})(0.250 \text{ m})^2 + 2\frac{1}{12}(0.035 \text{ kg})(0.500 \text{ m})^2 = 0.0102 \text{ kg} \cdot \text{m}^2$$

The torque is

$$|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = IAB \sin 90^\circ = (2.00 \text{ A})(0.500 \text{ m})(1.00 \text{ m})(3.00 \text{ T}) = 3.00 \text{ N} \cdot \text{m}$$

Using the above values, the rotational form of Newton's second law gives

$$\alpha = \frac{\tau}{I} = 290 \text{ rad/s}^2$$

**EVALUATE:** This angular acceleration will not continue because the torque changes as the coil turns.

**27.46. IDENTIFY:**  $\vec{\tau} = \vec{\mu} \times \vec{B}$  and  $U = -\mu B \cos \phi$ , where  $\mu = NIB$ .  $\tau = \mu B \sin \phi$ .

**SET UP:**  $\phi$  is the angle between  $\vec{B}$  and the normal to the plane of the loop.

**EXECUTE:** (a)  $\phi = 90^\circ$ .  $\diamond = NIAB \sin(90^\circ) = NIAB$ , direction  $\hat{k} \times \hat{j} = -\hat{i}$ .  $U = -\mu B \cos \phi = 0$ .

(b)  $\phi = 0$ .  $\diamond = NIAB \sin(0) = 0$ , no direction.  $U = -\mu B \cos \phi = -NIAB$ .

(c)  $\phi = 90^\circ$ .  $\diamond = NIAB \sin(90^\circ) = NIAB$ , direction  $-\hat{k} \times \hat{j} = \hat{i}$ .  $U = -\mu B \cos \phi = 0$ .

(d)  $\phi = 180^\circ$ :  $\diamond = NIAB \sin(180^\circ) = 0$ , no direction,  $U = -\mu B \cos(180^\circ) = NIAB$ .

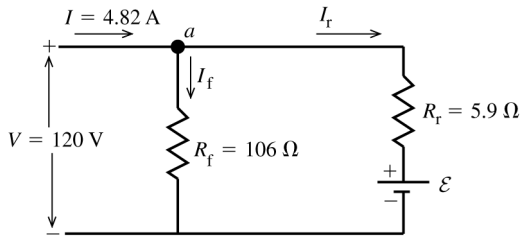
**EVALUATE:** When  $\tau$  is maximum,  $U = 0$ . When  $|U|$  is maximum,  $\tau = 0$ .

## 27.8. Moteur à courant continu

- Application des principes vus dans le chapitre.
- Fig. 27.39: Un moteur consiste en une boucle de courant (à laquelle on associe un moment magnétique) dans un champ  $\mathbf{B}$ , qui fera tourner la boucle en générant un moment de force. Il y a donc transfert d'énergie électrique en travail mécanique.
- Lire la p. 942 et l'exemple 27.11
- Autre exemple:

**27.49. IDENTIFY:** The circuit consists of two parallel branches with the potential difference of 120 V applied across each. One branch is the rotor, represented by a resistance  $R_r$  and an induced emf that opposes the applied potential. Apply the loop rule to each parallel branch and use the junction rule to relate the currents through the field coil and through the rotor to the 4.82 A supplied to the motor.

**SET UP:** The circuit is sketched in Figure 27.49.



$\mathcal{E}$  is the induced emf developed by the motor. It is directed so as to oppose the current through the rotor.

**Figure 27.49**

**EXECUTE:** (a) The field coils and the rotor are in parallel with the applied potential difference

$$V, \text{ so } V = I_f R_f. \quad I_f = \frac{V}{R_f} = \frac{120 \text{ V}}{106 \Omega} = 1.13 \text{ A}.$$

(b) Applying the junction rule to point  $a$  in the circuit diagram gives  $I - I_f - I_r = 0$ .

$$I_r = I - I_f = 4.82 \text{ A} - 1.13 \text{ A} = 3.69 \text{ A}.$$

(c) The potential drop across the rotor,  $I_r R_r + \mathcal{E}$ , must equal the applied potential difference

$$V : V = I_r R_r + \mathcal{E}$$

$$\mathcal{E} = V - I_r R_r = 120 \text{ V} - (3.69 \text{ A})(5.9 \Omega) = 98.2 \text{ V}$$

(d) The mechanical power output is the electrical power input minus the rate of dissipation of electrical energy in the resistance of the motor:

electrical power input to the motor

$$P_{\text{in}} = IV = (4.82 \text{ A})(120 \text{ V}) = 578 \text{ W}$$

electrical power loss in the two resistances

$$P_{\text{loss}} = I_f^2 R_f + I_r^2 R_r = (1.13 \text{ A})^2 (106 \Omega) + (3.69 \text{ A})^2 (5.9 \Omega) = 216 \text{ W}$$

mechanical power output

$$P_{\text{out}} = P_{\text{in}} - P_{\text{loss}} = 578 \text{ W} - 216 \text{ W} = 362 \text{ W}$$

The mechanical power output is the power associated with the induced emf  $\mathcal{E}$

$$P_{\text{out}} = P_{\mathcal{E}} = \mathcal{E} I_r = (98.2 \text{ V})(3.69 \text{ A}) = 362 \text{ W}, \text{ which agrees with the above calculation.}$$

**EVALUATE:** The induced emf reduces the amount of current that flows through the rotor. This motor differs from the one described in Example 27.12. In that example the rotor and field coils are connected in series and in this problem they are in parallel.