## Chapitre 28 - Sources de champ magnétique [8 au 10 juin]

DEVOIR : 28.8; 28.10; 28.29; 28.34; 28.38

### 28.1. Champ magnétique de charges en mouvement

- Eq. (28.2) et Fig. 28.1 : champ magnétique B au point $P$ par une charge $q$ qui se déplace à vitesse $\mathbf{v}$. Le vecteur $\mathbf{r}$ va de la charge au point $P$.
- $\quad$ Remarquer la relation (28.4)
- Lire l'exemple 28.1
- Autres exemples:
28.1. Identify and Set Up: Use Eq.(28.2) to calculate $\overrightarrow{\boldsymbol{B}}$ at each point.
$\vec{B}=\frac{\mu_{0}}{4 \pi} \frac{q \vec{v} \times \hat{r}}{r^{2}}=\frac{\mu_{0}}{4 \pi} \frac{q \vec{v} \times \vec{r}}{r^{3}}$, since $\hat{r}=\frac{\vec{r}}{r}$.
$\overrightarrow{\boldsymbol{v}}=\left(8.00 \times 10^{6} \mathrm{~m} / \mathrm{s}\right) \hat{\boldsymbol{j}}$ and $\overrightarrow{\boldsymbol{r}}$ is the vector from the charge to the point where the field is calculated.
Execute: (a) $\vec{r}=(0.500 \mathrm{~m}) \hat{i}, r=0.500 \mathrm{~m}$
$\vec{v} \times \vec{r}=v \hat{j} \times \hat{i}=-v r \hat{k}$
$\overrightarrow{\boldsymbol{B}}=-\frac{\mu_{0}}{4 \pi} \frac{q v}{r^{2}} \hat{\boldsymbol{k}}=-\left(1 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right) \frac{\left(6.00 \times 10^{-6} \mathrm{C}\right)\left(8.00 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)}{(0.500 \mathrm{~m})^{2}} \hat{\boldsymbol{k}}$
$\overrightarrow{\boldsymbol{B}}=-\left(1.92 \times 10^{-5} \mathrm{~T}\right) \hat{\boldsymbol{k}}$
(b) $\overrightarrow{\boldsymbol{r}}=-(0.500 \mathrm{~m}) \hat{\boldsymbol{j}}, r=0.500 \mathrm{~m}$
$\overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{r}}=-v \hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}}=0$ and $\overrightarrow{\boldsymbol{B}}=0$.
(c) $\overrightarrow{\boldsymbol{r}}=(0.500 \mathrm{~m}) \hat{\boldsymbol{k}}, r=0.500 \mathrm{~m}$
$\overrightarrow{\boldsymbol{v}} \times \vec{r}=v \hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}}=v r \hat{\boldsymbol{i}}$
$\overrightarrow{\boldsymbol{B}}=\left(1 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right) \frac{\left(6.00 \times 10^{-6} \mathrm{C}\right)\left(8.00 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)}{(0.500 \mathrm{~m})^{2}} \hat{\boldsymbol{i}}=+\left(1.92 \times 10^{-5} \mathrm{~T}\right) \hat{\boldsymbol{i}}$
(d) $\vec{r}=-(0.500 \mathrm{~m}) \hat{\boldsymbol{j}}+(0.500 \mathrm{~m}) \hat{\boldsymbol{k}}, r=\sqrt{(0.500 \mathrm{~m})^{2}+(0.500 \mathrm{~m})^{2}}=0.7071 \mathrm{~m}$
$\overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{r}}=v(0.500 \mathrm{~m})(-\hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}}+\hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}})=\left(4.00 \times 10^{6} \mathrm{~m}^{2} / \mathrm{s}\right) \hat{\boldsymbol{i}}$
$\overrightarrow{\boldsymbol{B}}=\left(1 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right) \frac{\left(6.00 \times 10^{-6} \mathrm{C}\right)\left(4.00 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)}{(0.7071 \mathrm{~m})^{3}} \hat{\boldsymbol{i}}=+\left(6.79 \times 10^{-6} \mathrm{~T}\right) \hat{\boldsymbol{i}}$
Evaluate: At each point $\overrightarrow{\boldsymbol{B}}$ is perpendicular to both $\overrightarrow{\boldsymbol{v}}$ and $\overrightarrow{\boldsymbol{r}} . B=0$ along the direction of $\overrightarrow{\boldsymbol{v}}$.
28.3. Identify: A moving charge creates a magnetic field.

SET UP: The magnetic field due to a moving charge is $B=\frac{\mu_{0}}{4 \pi} \frac{q v \sin \phi}{r^{2}}$.
Execute: Substituting numbers into the above equation gives
(a) $B=\frac{\mu_{0}}{4 \pi} \frac{q v \sin \phi}{r^{2}}=\frac{4 \pi \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}}{4 \pi} \frac{\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(3.0 \times 10^{7} \mathrm{~m} / \mathrm{s}\right) \sin 30^{\circ}}{\left(2.00 \times 10^{-6} \mathrm{~m}\right)^{2}}$.
$B=6.00 \times 10^{-8} \mathrm{~T}$, out of the paper, and it is the same at point $B$.
(b) $B=\left(1.00 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(3.00 \times 10^{7} \mathrm{~m} / \mathrm{s}\right) /\left(2.00 \times 10^{-6} \mathrm{~m}\right)^{2}$
$B=1.20 \times 10^{-7} \mathrm{~T}$, out of the page.
(c) $B=0 \mathrm{~T}$ since $\sin \left(180^{\circ}\right)=0$.

Evaluate: Even at high speeds, these charges produce magnetic fields much less than the Earth's magnetic field.
28.6. Identify: Apply $\overrightarrow{\boldsymbol{B}}=\frac{q \overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{r}}}{4} \frac{\text { For the magnetic force, apply the results of Example 28.1, except }}{r^{3}}$. here the two charges and velocities are different.
SET UP: In part (a), $r=d$ and $\overrightarrow{\boldsymbol{r}}$ is perpendicular to $\overrightarrow{\boldsymbol{v}}$ in each case, so $\frac{|\overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{r}}|}{r^{3}}=\frac{v}{d^{2}}$. For calculating the force between the charges, $r=2 d$.
EXECUTE: (a) $B_{\text {total }}=B+B^{\prime}=\frac{\mu_{0}}{4 \pi}\left(\frac{q v}{d^{2}}+\frac{q^{\prime} v^{\prime}}{d^{2}}\right)$.
$B=\frac{\omega_{0}}{4 \pi}\left(\frac{\left(8.0 \times 10^{-6} \mathrm{C}\right)\left(4.5 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)}{(0.120 \mathrm{~m})^{2}}+\frac{\left(3.0 \times 10^{-6} \mathrm{C}\right)\left(9.0 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)}{(0.120 \mathrm{~m})^{2}}\right)=4.38 \times 10^{-4} \mathrm{~T}$.
The direction of $\overrightarrow{\boldsymbol{B}}$ is into the page.
(b) Following Example 28.1 we can find the magnetic force between the charges:
$F_{B}=\frac{q q^{\prime} v v^{\prime}}{4}=\left(10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right) \frac{\left(8.00 \times 10^{-6} \mathrm{C}\right)\left(3.00 \times 10^{-6} \mathrm{C}\right)\left(4.50 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)\left(9.00 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)}{(0.240 \mathrm{~m})^{2}}$
$F_{B}=1.69 \times 10^{-3} \mathrm{~N}$. The force on the upper charge points up and the force on the lower charge points down. The Coulomb force between the charges is
$F_{\mathrm{C}}=k \frac{q_{1} q_{2}}{r^{2}}=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(8.0 \times 10^{-6} \mathrm{C}\right)\left(3.0 \times 10^{-6} \mathrm{C}\right)}{(0.240 \mathrm{~m})^{2}}=3.75 \mathrm{~N}$. The force on the upper charge points up and the force on the lower charge points down. The ratio of the Coulomb force to the magnetic force is $\frac{F_{\mathrm{C}}}{F_{B}}=\frac{c^{2}}{v_{1} v_{2}}=\frac{3.75 \mathrm{~N}}{1.69 \times 10^{-3} \mathrm{~N}}=2.22 \times 10^{3}$; the Coulomb force is much larger.
(b) The magnetic forces are reversed in direction when the direction of only one velocity is reversed but the magnitude of the force is unchanged.
Evaluate: When two charges have the same sign and move in opposite directions, the force between them is repulsive. When two charges of the same sign move in the same direction, the force between them is attractive.

### 28.2. Loi de Biot-Savart

- Eq. (28.6) : la loi de Biot-Savart donne le champ B créé par un élément de courant $I$ (longueur $\mathrm{d} \mathbf{l}$ ) à un point $P$. Le vecteur $\mathbf{r}$ va de l'élément de courant jusqu'au point $P$. L'Eq. (28.7) donne le champ créé par un fil de longueur finie.
- P. 961: lire la partie Current Elements - Magnetic Field Lines
- Lire l'exemple 28.2
- Autres exemples :
28.11. Identify and Set Up: The magnetic field produced by an infinitesimal current element is given by Eq.(28.6).
$d \overrightarrow{\boldsymbol{B}}=\frac{\mu_{0}}{4 \pi} \frac{\vec{l} \times \hat{r}}{r^{2}}$ As in Example 28.2 use this equation for the finite $0.500-\mathrm{mm}$ segment of wire since the $\Delta l=0.500 \mathrm{~mm}$ length is much smaller than the distances to the field points.
$\overrightarrow{\boldsymbol{B}}=\frac{\mu_{0}}{4 \pi} \frac{I \Delta \overrightarrow{\boldsymbol{l}} \times \hat{r}}{r^{2}}=\frac{\mu_{0}}{4 \pi} \frac{I \Delta \overrightarrow{\boldsymbol{l}} \times \vec{r}}{r^{3}}$
$I$ is in the $+z$-direction, so $\Delta \overrightarrow{\boldsymbol{l}}=\left(0.500 \times 10^{-3} \mathrm{~m}\right) \hat{\boldsymbol{k}}$
Execute: (a) Field point is at $x=2.00 \mathrm{~m}, y=0, z=0$ so the vector $\overrightarrow{\boldsymbol{r}}$ from the source point (at the origin) to the field point is $\overrightarrow{\boldsymbol{r}}=(2.00 \mathrm{~m}) \hat{\boldsymbol{i}}$.

$$
\begin{aligned}
& \Delta \overrightarrow{\boldsymbol{l}} \times \overrightarrow{\boldsymbol{r}}=\left(0.500 \times 10^{-3} \mathrm{~m}\right)(2.00 \mathrm{~m}) \hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}}=+\left(1.00 \times 10^{-3} \mathrm{~m}^{2}\right) \hat{\boldsymbol{j}} \\
& \overrightarrow{\boldsymbol{B}}=\frac{\left(1 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right)(4.00 \mathrm{~A})\left(1.00 \times 10^{-3} \mathrm{~m}^{2}\right)}{(2.00 \mathrm{~m})^{3}} \hat{\boldsymbol{j}}=\left(5.00 \times 10^{-11} \mathrm{~T}\right) \hat{\boldsymbol{j}}
\end{aligned}
$$

(b) $\overrightarrow{\boldsymbol{r}}=(2.00 \mathrm{~m}) \hat{\boldsymbol{j}}, r=2.00 \mathrm{~m}$.
$\Delta \overrightarrow{\boldsymbol{l}} \times \vec{r}=\left(0.500 \times 10^{-3} \mathrm{~m}\right)(2.00 \mathrm{~m}) \hat{\boldsymbol{k}} \times \hat{\boldsymbol{j}}=-\left(1.00 \times 10^{-3} \mathrm{~m}^{2}\right) \hat{\boldsymbol{i}}$
$\overrightarrow{\boldsymbol{B}}=-\frac{\left(1 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right)(4.00 \mathrm{~A})\left(1.00 \times 10^{-3} \mathrm{~m}^{2}\right)}{(2.00 \mathrm{~m})^{3}} \hat{\boldsymbol{i}}=-\left(5.00 \times 10^{-11} \mathrm{~T}\right) \hat{\boldsymbol{i}}$
(c) $\overrightarrow{\boldsymbol{r}}=(2.00 \mathrm{~m})(\hat{\boldsymbol{i}}+\hat{\boldsymbol{j}}), r=\sqrt{2}(2.00 \mathrm{~m})$.
$\Delta \overrightarrow{\boldsymbol{l}} \times \overrightarrow{\boldsymbol{r}}=\left(0.500 \times 10^{-3} \mathrm{~m}\right)(2.00 \mathrm{~m}) \hat{\boldsymbol{k}} \times(\hat{\boldsymbol{i}}+\hat{\boldsymbol{j}})=\left(1.00 \times 10^{-3} \mathrm{~m}^{2}\right)(\hat{\boldsymbol{j}}-\hat{\boldsymbol{i}})$
$\overrightarrow{\boldsymbol{B}}=\frac{\left(1 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right)(4.00 \mathrm{~A})\left(1.00 \times 10^{-3} \mathrm{~m}^{2}\right)}{[\sqrt{2}(2.00 \mathrm{~m})]^{3}}(\hat{\boldsymbol{j}}-\hat{\boldsymbol{i}})=\left(-1.77 \times 10^{-11} \mathrm{~T}\right)(\hat{\boldsymbol{i}}-\hat{\boldsymbol{j}})$
(d) $\overrightarrow{\boldsymbol{r}}=(2.00 \mathrm{~m}) \hat{\boldsymbol{k}}, r=2.00 \mathrm{~m}$.
$\Delta \overrightarrow{\boldsymbol{l}} \times \overrightarrow{\boldsymbol{r}}=\left(0.500 \times 10^{-3} \mathrm{~m}\right)(2.00 \mathrm{~m}) \hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}}=0 ; \overrightarrow{\boldsymbol{B}}=0$.
Evaluate: At each point $\overrightarrow{\boldsymbol{B}}$ is perpendicular to both $\overrightarrow{\boldsymbol{r}}$ and $\Delta \overrightarrow{\boldsymbol{l}} . B=0$ along the length of the wire.
28.12. Identify: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $d B=\frac{\mu_{0}}{4 \pi} \frac{I d l \sin \phi}{r^{2}}$.
Both fields are into the page, so their magnitudes add.
ExECUTE: Applying the law of Biot and Savart for the 12.0-A current gives

$$
d B=\frac{4 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}}{4} \frac{(12.0 \mathrm{~A})(0.00150 \mathrm{~m})\left(\frac{2.50 \mathrm{~cm}}{8.00 \mathrm{~cm}}\right)}{(0.0800 \mathrm{~m})^{2}}=8.79 \times 10^{-8} \mathrm{~T}
$$

The field from the 24.0 -A segment is twice this value, so the total field is $2.64 \times 10^{-7} \mathrm{~T}$, into the page.
Evaluate: The rest of each wire also produces field at $P$. We have calculated just the field from the two segments that are indicated in the problem.
28.13. Identify: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $d B=\frac{\mu_{0}}{4 \pi} \frac{I d l \sin \phi}{r^{2}}$. Both fields are into the page, so their magnitudes add.
Execute: Applying the Biot and Savart law, where $r=\frac{1}{2} \sqrt{(3.00 \mathrm{~cm})^{2}+(3.00 \mathrm{~cm})^{2}}=2.121 \mathrm{~cm}$, we have

$$
d B=2 \frac{4 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}}{4} \frac{(28.0 \mathrm{~A})(0.00200 \mathrm{~m}) \sin 45.0^{\circ}}{(0.02121 \mathrm{~m})^{2}}=1.76 \times 10^{-5} \mathrm{~T}, \text { into the paper. }
$$

Evaluate: Even though the two wire segments are at right angles, the magnetic fields they create are in the same direction.

### 28.3. Champ magnétique par un fil conducteur droit

- Eq. (28.9) : B à distance $r$ d'un conducteur long
- Lire les exemples 28.3 et 28.4
- Autres exemples:
28.19. Identify: The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.
SET UP: For the wire, $B_{\text {wire }}=\frac{\mu_{0} I}{2 \pi r}$ and the direction of $B_{\text {wire }}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook. $\overrightarrow{\boldsymbol{B}}_{0}=\left(1.50 \times 10^{-6} \mathrm{~T}\right) \hat{\boldsymbol{i}}$.
EXECUTE: (a) At $(0,0,1 \mathrm{~m}), \overrightarrow{\boldsymbol{B}}=\overrightarrow{\boldsymbol{B}}_{0}-\frac{\mu_{0} I}{2 \pi r} \hat{\boldsymbol{i}}=\left(1.50 \times 10^{-6} \mathrm{~T}\right) \hat{\boldsymbol{i}}-\frac{\mu_{0}(8.00 \mathrm{~A})}{2 \pi(1.00 \mathrm{~m})} \hat{\boldsymbol{i}}=-\left(1.0 \times 10^{-7} \mathrm{~T}\right) \hat{\boldsymbol{i}}$.
(b) At $(1 \mathrm{~m}, 0,0), \overrightarrow{\boldsymbol{B}}=\overrightarrow{\boldsymbol{B}}_{0}+\frac{{ }_{0} I}{2 r} \hat{\boldsymbol{k}}=\left(1.50 \times 10^{-6} \mathrm{~T}\right) \hat{\boldsymbol{i}}+\frac{{ }_{0}(8.00 \mathrm{~A})}{2(1.00 \mathrm{~m})} \hat{\boldsymbol{k}}$.
$\overrightarrow{\boldsymbol{B}}=\left(1.50 \times 10^{-6} \mathrm{~T}\right) \hat{\boldsymbol{i}}+\left(1.6 \times 10^{-6} \mathrm{~T}\right) \hat{\boldsymbol{k}}=2.19 \times 10^{-6} \mathrm{~T}$, at $=46.8^{\circ}$ from $x$ to $z$.
(c) At $(0,0,-0.25 \mathrm{~m}), \overrightarrow{\boldsymbol{B}}=\overrightarrow{\boldsymbol{B}}_{0}+\frac{\mu_{0} I}{2 r} \hat{\boldsymbol{i}}=\left(1.50 \times 10^{-6} \mathrm{~T}\right) \hat{\boldsymbol{i}}+\frac{(8.00 \mathrm{~A})}{2(0.25 \mathrm{~m})} \hat{\boldsymbol{i}}=\left(7.9 \times 10^{-6} \mathrm{~T}\right) \hat{\boldsymbol{i}}$.

Evaluate: At point $c$ the two fields are in the same direction and their magnitudes add. At point $a$ they are in opposite directions and their magnitudes subtract. At point $b$ the two fields are perpendicular.
28.24. Identify: Use Eq.(28.9) and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.
SET UP: The three known currents are shown in Figure 28.24.


$$
\begin{aligned}
& \overrightarrow{\boldsymbol{B}}_{1} \otimes, \overrightarrow{\boldsymbol{B}}_{2} \otimes, \overrightarrow{\boldsymbol{B}}_{3} \odot \\
& B=\frac{\mu_{0} I}{2 \pi r} ; r=0.200 \mathrm{~m} \text { for each wire }
\end{aligned}
$$

Figure 28.24
Execute: Let $\odot$ be the positive $z$-direction. $I_{1}=10.0 \mathrm{~A}, I_{2}=8.0 \mathrm{~A}, I_{3}=20.0 \mathrm{~A}$. Then $B_{1}=1.00 \times 10^{-5} \mathrm{~T}, B_{2}=0.80 \times 10^{-5} \mathrm{~T}$, and $B_{3}=2.00 \times 10^{-5} \mathrm{~T}$.
$B_{1 \mathrm{z}}=-1.00 \times 10^{-5} \mathrm{~T}, B_{2 \mathrm{z}}=-0.80 \times 10^{-5} \mathrm{~T}, B_{3 \mathrm{z}}=+2.00 \times 10^{-5} \mathrm{~T}$
$B_{1 z}+B_{2 z}+B_{3 z}+B_{4 z}=0$
$B_{4 z}=-\left(B_{1 z}+B_{2 z}+B_{3 z}\right)=-2.0 \times 10^{-6} \mathrm{~T}$
To give $\overrightarrow{\boldsymbol{B}}_{4}$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.
$B_{4}=\frac{\mu_{0} I}{2 \pi r}$ so $I_{4}=\frac{r B_{4}}{\left(\mu_{0} / 2 \pi\right)}=\frac{(0.200 \mathrm{~m})\left(2.0 \times 10^{-6} \mathrm{~T}\right)}{\left(2 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right)}=2.0 \mathrm{~A}$

Evaluate: The fields of wires \#2 and \#3 are in opposite directions and their net field is the same as due to a current $20.0 \mathrm{~A}-8.0 \mathrm{~A}=12.0 \mathrm{~A}$ in one wire. The field of wire \#4 must be in the same direction as that of wire $\# 1$, and $10.0 \mathrm{~A}+I_{4}=12.0 \mathrm{~A}$.

### 28.4. Force entre conducteurs parallèles

- Eq. (28.11) : Force entre deux fils longs parallèles. Obtenue des Eqs. (27.17) et (28.9).
- Lire l'exemple 28.5
- Autres exemples :
28.26. Identify: Apply Eq.(28.11).

SET Up: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.
EXECUTE: (a) $\frac{F}{L}=\frac{\mu_{0} I_{1} I_{2}}{2 \pi r}$ gives $I_{2}=\frac{F}{L} \frac{2 \pi r}{\mu_{0} I_{1}}=\left(4.0 \times 10^{-5} \mathrm{~N} / \mathrm{m}\right) \frac{2 \pi(0.0250 \mathrm{~m})}{\mu_{0}(0.60 \mathrm{~A})}=8.33 \mathrm{~A}$.
(b) The two wires repel so the currents are in opposite directions.

Evaluate: The force between the two wires is proportional to the product of the currents in the wires.
28.28. Identify: Apply Eq.(28.11) for the force from each wire.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.
EXECUTE: On the top wire $\frac{F}{L}=\frac{\mu_{0} I^{2}}{2 \pi}\left(\frac{1}{d}-\frac{1}{2 d}\right)=\frac{\mu_{0} I^{2}}{4 \pi d}$, upward. On the middle wire, the magnetic forces cancel so the net force is zero. On the bottom wire $\frac{F}{L}=\frac{\mu_{0} I^{2}}{2 \pi}\left(-\frac{1}{d}+\frac{1}{2 d}\right)=\frac{\mu_{0} I^{2}}{4 \pi d}$, downward.
Evaluate: The net force on the middle wire is zero because at the location of the middle wire the net magnetic field due to the other two wires is zero.

### 28.5. Champ magnétique d'une boucle de courant circulaire

- Eq. (28.15) : donne $B_{x}$ sur l'axe de la boucle circulaire. Obtenu en intégrant la loi de Biot-Savart. La composante $y$ est nulle par symétrie.
- Eq. (28.17) donne $B$ au centre de la boucle. Obtenue de Eq. (28.15) avec $x=0$.
- Voir exemple 28.6.
- Autres exemples:
28.30. IdENTIFY: The magnetic field at the center of a circular loop is $B=\frac{\mu_{0} I}{2 R}$. By symmetry each segment of the loop that has length $\Delta l$ contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.
SET UP: Since the straight sections produce no field at $P$, the field at $P$ is $B=\frac{\mu_{0} I}{4 R}$.
ExECUTE: $\quad B=\frac{\mu_{0} I}{4 R}$. The direction of $\overrightarrow{\boldsymbol{B}}$ is given by the right-hand rule: $\overrightarrow{\boldsymbol{B}}$ is directed into the page.

Evaluate: For a quarter-circle section of wire the magnetic field at its center of curvature is $B=\frac{\mu_{0} I}{8 R}$.
28.32. Identify: Apply Eq.(28.16).

SET UP: At the center of the coil, $x=0 . a$ is the radius of the coil, 0.0240 m .
EXECUTE: (a) $B_{x}=N I / 2 a$, so $I=\frac{2 a B_{x}}{{ }_{0} N}=\frac{2(0.024 \mathrm{~m})(0.0580 \mathrm{~T})}{\left(4 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right)(800)}=2.77 \mathrm{~A}$
(b) At the center, $B_{\mathrm{c}}=\mu_{0} N I / 2 a$. At a distance $x$ from the center,
$B_{x}=\frac{\mu_{0} N I a^{2}}{2\left(x^{2}+a^{2}\right)^{3 / 2}}=\left(\frac{\mu_{0} N I}{2 a}\right)\left(\frac{a^{3}}{\left(x^{2}+a^{2}\right)^{3 / 2}}\right)=B_{\mathrm{c}}\left(\frac{a^{3}}{\left(x^{2}+a^{2}\right)^{3 / 2}}\right) . \quad B_{x}=\frac{1}{2} B_{\mathrm{c}}$ says $\frac{a^{3}}{\left(x^{2}+a^{2}\right)^{3 / 2}}=\frac{1}{2}$, and
$\left(x^{2}+a^{2}\right)^{3}=4 a^{6}$. Since $a=0.024 \mathrm{~m}, x=0.0184 \mathrm{~m}$.
Evaluate: As shown in Figure 28.41 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

### 28.6. Loi d'Ampère

- Eq. (28.20) : La loi d'Ampère est analogue à la loi de Gauss (Eq. (22.8), qui permet parfois de calculer $\mathbf{E}$, sachant que le flux est proportionnel à la charge). Ici, l'intégrale de ligne (ou circulation) de $\mathbf{B}$ est proportionnelle au courant contenu dans le parcours; cette relation permet parfois de calculer B.
- Exemples :
28.35. Identify: Apply Ampere's law.

SET UP: $\quad \mu_{0}=4 \pi \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}$
ExEcute: (a) $\dot{U} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{l}}=I_{\text {encl }}=3.83 \times 10^{-4} \mathrm{~T} \cdot \mathrm{~m}$ and $I_{\text {encl }}=305 \mathrm{~A}$.
(b) $-3.83 \times 10^{-4} \mathrm{~T} \cdot \mathrm{~m}$ since at each point on the curve the direction of $d \overrightarrow{\boldsymbol{l}}$ is reversed.

Evaluate: The line integral $\mathbf{U} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{l}}$ around a closed path is proportional to the net current that is enclosed by the path.
28.36. Identify: Apply Ampere's law.

SET UP: From the right-hand rule, when going around the path in a counterclockwise direction currents out of the page are positive and currents into the page are negative.
ExECUTE: Path a: $I_{\text {encl }}=0 \Rightarrow \hat{U} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{l}}=0$.
Path b: $I_{\text {encl }}=-I_{1}=-4.0 \mathrm{~A} \Rightarrow \hat{\mathbf{U}} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{l}}=-{ }_{0}(4.0 \mathrm{~A})=-5.03 \times 10^{-6} \mathrm{~T} \cdot \mathrm{~m}$.
Path c: $I_{\text {encl }}=-I_{1}+I_{2}=-4.0 \mathrm{~A}+6.0 \mathrm{~A}=2.0 \mathrm{~A} \Rightarrow \mathbf{U} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{l}}={ }_{0}(2.0 \mathrm{~A})=2.51 \times 10^{-6} \mathrm{~T} \cdot \mathrm{~m}$
Path d: $I_{\text {encl }}=-I_{1}+I_{2}+I_{3}=4.0 \mathrm{~A} \Rightarrow \hat{U} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{l}}=+(4.0 \mathrm{~A})=5.03 \times 10^{-6} \mathrm{~T} \cdot \mathrm{~m}$.
Evaluate: If we instead went around each path in the clockwise direction, the sign of the line integral would be reversed.

### 28.7. Applications de la loi d'Ampère

- Lire les exemples de cette section
- Autres exemples :
28.37. Identify: Apply Ampere's law.

SET UP: To calculate the magnetic field at a distance $r$ from the center of the cable, apply Ampere's law to a circular path of radius $r$. By symmetry, $\mathbf{U} \overrightarrow{\mathbf{B}} \cdot d \overrightarrow{\boldsymbol{l}}=B(2 \pi r)$ for such a path.
EXECUTE: (a) For $a<r<b, I_{\text {encl }}=I \Rightarrow \tilde{U} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{l}}={ }_{0} I \Rightarrow B 2 r={ }_{0} I \Rightarrow B=\frac{{ }_{0} I}{2 r}$.
(b) For $r>c$, the enclosed current is zero, so the magnetic field is also zero.

Evaluate: A useful property of coaxial cables for many applications is that the current carried by the cable doesn't produce a magnetic field outside the cable.
28.40. IdENTIFY: $B={ }_{0} n I=\frac{{ }_{0} N I}{L}$

Set Up: $\quad L=0.150 \mathrm{~m}$
ExECUTE: $\quad B=\frac{{ }_{0}(600)(8.00 \mathrm{~A})}{(0.150 \mathrm{~m})}=0.0402 \mathrm{~T}$
Evaluate: The field near the center of the solenoid is independent of the radius of the solenoid, as long as the radius is much less than the length.
28.42. IDENTIFY and SET UP: At the center of a long solenoid $B=\mu_{0} n I=\mu_{0} \frac{N}{L} I$.

EXECUTE: $\quad I=\frac{B L}{\mu_{0} N}=\frac{(0.150 \mathrm{~T})(1.40 \mathrm{~m})}{\left(4 \pi \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right)(4000)}=41.8 \mathrm{~A}$
Evaluate: The magnetic field inside the solenoid is independent of the radius of the solenoid, if the radius is much less than the length, as is the case here.
28.44. IDENTIFY: Example 28.10 shows that outside a toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B=\frac{\omega_{0} N I}{2 r}$.

SET UP: The torus extends from $r_{1}=15.0 \mathrm{~cm}$ to $r_{2}=18.0 \mathrm{~cm}$.
ExECUTE: (a) $r=0.12 \mathrm{~m}$, which is outside the torus, so $B=0$.
(b) $r=0.16 \mathrm{~m}$, so $B=\frac{\mu_{0} N I}{2 \pi r}=\frac{\mu_{0}(250)(8.50 \mathrm{~A})}{2 \pi(0.160 \mathrm{~m})}=2.66 \times 10^{-3} \mathrm{~T}$.
(c) $r=0.20 \mathrm{~m}$, which is outside the torus, so $B=0$.

Evaluate: The magnetic field inside the torus is proportional to $1 / r$, so it varies somewhat over the cross-section of the torus.

