Chapitre 29 – Induction électromagnétique [13 au 15 juin]

DEVOIR : 29.8; 29.20; 29.22; 29.30; 29.36

29.1. Expériences d'induction

- Il n’est pas nécessaire de lire cette section.
- Ce qu’il faut retenir de la Fig. 29.1, c’est que, quand on déplace l’aimant (ou une seconde bobine), c’est l’intensité de **B** dans la bobine initiale.

29.2. Loi de Faraday

- Eq. (29.3) est la loi de Faraday. Le signe sera expliqué après l’exemple 29.1.
- Lire l’exemple 29.1
- P. 997: Lire *Direction of Induced EMF* et consulter la Fig. 29.6
- Eq. (29.4) : N est le nombre d’enroulements
- Lire les exemples 29.2, 4, 5, 6 et 7
- Autres exemples:

29.7. **IDENTIFY:** Calculate the flux through the loop and apply Faraday’s law.

SET UP: To find the total flux integrate $d\Phi_B$ over the width of the loop. The magnetic field of a long straight wire, at distance r from the wire, is $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** When $B = \dfrac{\mu_0 i}{2\pi r}$, into the page.

(b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r} Ldr.$

(c) $\Phi_B = \int_a^b d\Phi_B = \dfrac{\mu_0 iL}{2\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 iL}{2\pi}\ln(b/a).$

(d) $\mathcal{E} = \dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 L}{2\pi}\ln(b/a)\dfrac{di}{dt}.$

(e) $\mathcal{E} = \dfrac{\mu_0(0.240\text{ m})}{2\pi}\ln(0.360/0.120)(9.60\text{ A/s}) = 5.06\times 10^{-7}\text{ V}.$

EVALUATE: The induced emf is proportional to the rate at which the current in the long straight wire is changing

29.9. **IDENTIFY** and **SET UP:** Use Faraday’s law to calculate the emf (magnitude and direction). The direction of the induced current is the same as the direction of the emf. The flux changes because the area of the loop is changing; relate dA/dt to dc/dt, where c is the circumference of the loop.

(a) EXECUTE: $c = 2\pi r$ and $A = \pi r^2$ so $A = c^2/4\pi$

$\Phi_B = BA = (B/4\pi)c^2$

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \left(\frac{B}{2\pi}\right)c\left|\frac{dc}{dt}\right|$$

At $t = 9.0$ s, $c = 1.650\text{ m} - (9.0\text{ s})(0.120\text{ m/s}) = 0.570\text{ m}$

$|\mathcal{E}| = (0.500\text{ T})(1/2\pi)(0.570\text{ m})(0.120\text{ m/s}) = 5.44\text{ mV}$

(b) SET UP: The loop and magnetic field are sketched in Figure 29.9.

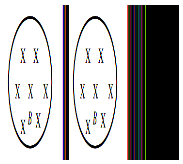

Figure 29.9

Take into the page to be the positive direction for $\vec{A}$. Then the magnetic flux is positive.

EXECUTE: The positive flux is decreasing in magnitude; $d\Phi_B / dt$ is negative and $\mathcal{E}$ is positive. By the right-hand rule, for $\vec{A}$ into the page, positive $\mathcal{E}$ is clockwise.

EVALUATE: Even though the circumference is changing at a constant rate, dA/dt is not constant and $|\mathcal{E}|$ is not constant. Flux $\otimes$ is decreasing so the flux of the induced current is $\otimes$ and this means that I is clockwise, which checks.

29.10. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.
SET UP: The flux through a coil is $\Phi = NBA \cos \phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.
EXECUTE: **(a)** and **(c)** The magnetic flux is constant, so the induced emf is zero.
(b) The area inside the field is changing. If we let x be the length (along the 30.0-cm side) in the field, then
$A = (0.400 \text{ m})x$. $\Phi_B = BA = (0.400 \text{ m})x$

$$\mathcal{E} = d\Phi/dt = B\, d[(0.400 \text{ m})x]/dt = B(0.400 \text{ m})dx/dt = B(0.400 \text{ m})v$$

$$\mathcal{E} = (1.25 \text{ T})(0.400 \text{ m})(0.0200 \text{ m/s}) = 0.0100 \text{ V}$$

EVALUATE: It is not a large *flux* that induces an emf, but rather a large *rate of change* of the flux. The induced emf in part (b) is small enough to be ignored in many instances.

29.12. **IDENTIFY:** Use the results of Example 29.5.

SET UP: $\mathcal{E}_{max} = NBA\omega$. $\mathcal{E}_{av} = \frac{2}{\pi}\mathcal{E}_{max}$. $\omega = (440 \text{ rev/min})\left(\frac{2\pi \text{ rad/rev}}{60 \text{ s/min}}\right) = 46.1 \text{ rad/s}$.

EXECUTE: **(a)** $\mathcal{E}_{max} = NBA\omega = (150)(0.060 \text{ T})\pi(0.025 \text{ m})^2(46.1 \text{ rad/s}) = 0.814 \text{ V}$

(b) $\mathcal{E}_{av} = \frac{2}{\pi}\mathcal{E}_{max} = \frac{2}{\pi}(0.815 \text{ V}) = 0.519 \text{ V}$

EVALUATE: In $\mathcal{E}_{max} = NBA\omega$, ω must be in rad/s.

29.3. Loi de Lenz

- La loi de Lenz est une autre façon de déterminer la direction de la fém
- Voir l'encadré jaune de la P. 1004
- "cause of the effect" veut dire "la variation de flux"
- La loi de Lenz découle essentiellement de la conservation de l'énergie, comme expliqué dans le paragraphe précédant l'exemple 29.8
- Lire les exemples 29.8 et 29.9
- Autres exemples :

29.16. **IDENTIFY:** By Lenz's law, the induced current flows to oppose the flux change that caused it.
SET UP and **EXECUTE:** The magnetic field is outward through the round coil and is decreasing, so the magnetic field due to the induced current must also point outward to oppose this decrease. Therefore the induced current is counterclockwise.
EVALUATE: Careful! Lenz's law does not say that the induced current flows to oppose the magnetic flux. Instead it says that the current flows to oppose the *change* in flux.

29.18. **IDENTIFY:** Apply Lenz's law.

SET UP: The field of the induced current is directed to oppose the change in flux in the primary circuit.
EXECUTE: **(a)** The magnetic field in A is to the left and is increasing. The flux is increasing so the field due to the induced current in B is to the right. To produce magnetic field to the right, the induced current flows through R from right to left.
(b) The magnetic field in A is to the right and is decreasing. The flux is decreasing so the field due to the induced current in B is to the right. To produce magnetic field to the right the induced current flows through R from right to left.
(c) The magnetic field in A is to the right and is increasing. The flux is increasing so the field due to the induced current in B is to the left. To produce magnetic field to the left the induced current flows through R from left to right.
EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.4. Fém de mouvement

- Lire la section, y compris les deux exemples
- Eq. (29.7) décrit un conducteur de forme quelconque dans un champ **B**. Elle suit de $\varepsilon = \dfrac{W}{q} = \dfrac{1}{q}\int\left(\vec{F}_E + \vec{F}_B\right)\cdot d\vec{l} = \dfrac{1}{q}\int q\left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot d\vec{l}$ dans laquelle **E** = **0**.
- Autres exemples :

29.21. **IDENTIFY:** A conductor moving in a magnetic field may have a potential difference induced across it, depending on how it is moving.
SET UP: The induced emf is $\mathcal{E} = vBL\sin\phi$, where ϕ is the angle between the velocity and the magnetic field.
EXECUTE: **(a)** $\mathcal{E} = vBL\sin\phi = (5.00\text{ m/s})(0.450\text{ T})(0.300\text{ m})(\sin 90°) = 0.675\text{ V}$
(b) The positive charges are moved to end b, so b is at the higher potential.
(c) $E = V/L = (0.675\text{ V})/(0.300\text{ m}) = 2.25\text{ V/m}$. The direction of $\vec{E}$ is from, b to a.
(d) The positive charge are pushed to b, so b has an excess of positive charge.
(e) (i) If the rod has no appreciable thickness, $L = 0$, so the emf is zero. (ii) The emf is zero because no magnetic force acts on the charges in the rod since it moves parallel to the magnetic field.
EVALUATE: The motional emf is large enough to have noticeable effects in some cases.

29.25. **IDENTIFY** and **SET UP:** $\mathcal{E} = vBL$. Use Lenz's law to determine the direction of the induced current. The force F_{ext} required to maintain constant speed is equal and opposite to the force F_I that the magnetic field exerts on the rod because of the current in the rod.
EXECUTE: **(a)** $\mathcal{E} = vBL = (7.50\text{ m/s})(0.800\text{ T})(0.500\text{ m}) = 3.00\text{ V}$
(b) $\vec{B}$ is into the page. The flux increases as the bar moves to the right, so the magnetic field of the induced current is out of the page inside the circuit. To produce magnetic field in this direction the induced current must be counterclockwise, so from b to a in the rod.
(c) $I = \dfrac{\mathcal{E}}{R} = \dfrac{3.00\text{ V}}{1.50\ \Omega} = 2.00\text{ A}$. $F_I = ILB\sin\phi = (2.00\text{ A})(0.500\text{ m})(0.800\text{ T})\sin 90∞ = 0.800\text{ N}$. $\vec{F}_I$ is to the left. To keep the bar moving to the right at constant speed an external force with magnitude $F_{\text{ext}} = 0.800\text{ N}$ and directed to the right must be applied to the bar.
(d) The rate at which work is done by the force F_{ext} is $F_{\text{ext}}v = (0.800\text{ N})(7.50\text{ m/s}) = 6.00\text{ W}$. The rate at which thermal energy is developed in the circuit is $I^2R = (2.00\text{ A})(1.50\ \Omega) = 6.00\text{ W}$. These two rates are equal, as is required by conservation of energy.
EVALUATE: The force on the rod due to the induced current is directed to oppose the motion of the rod. This agrees with Lenz's law.

29.27. **IDENTIFY:** A bar moving in a magnetic field has an emf induced across its ends.

SET UP: The induced potential is $\mathcal{E} = vBL \sin \phi$.
EXECUTE: Note that $\phi = 90°$ in all these cases because the bar moved perpendicular to the magnetic field. But the effective length of the bar, $L \sin \theta$, is different in each case.
(a) $\mathcal{E} = vBL \sin \theta = (2.50 \text{ m/s})(1.20 \text{ T})(1.41 \text{ m}) \sin (37.0°) = 2.55 \text{ V}$, with a at the higher potential because positive charges are pushed toward that end.
(b) Same as (a) except $\theta = 53.0°$, giving 3.38 V, with a at the higher potential.
(c) Zero, since the velocity is parallel to the magnetic field.
(d) The bar must move perpendicular to its length, for which the emf is 4.23 V. For $V_b > V_a$, it must move upward and to the left (toward the second quadrant) perpendicular to its length.
EVALUATE: The orientation of the bar affects the potential induced across its ends.

29.5. Champs électriques induits

- À l'Eq. (28.20), on avait la circulation de **B** proportionnelle au courant. Pour le champ **E**, l'Eq. (29.10) nous montre que la circulation est proportionnelle au taux de variation du flux magnétique. (Nous verrons à l'Eq. (29.20) que quelque chose manque dans Eq. (28.20) qui rendra les équations semblables.)
- Lire la section et l'exemple 29.12.
- Autres exemples :

29.28. **IDENTIFY:** Use Eq.(29.10) to calculate the induced electric field E at a distance r from the center of the solenoid. Away from the ends of the solenoid, $B = \mu_0 nI$ inside and $B = 0$ outside.
(a) SET UP: The end view of the solenoid is sketched in Figure 29.28.

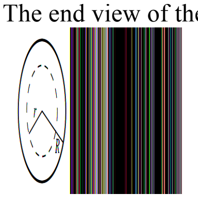

Figure 29.28

Let R be the radius of the solenoid.

Apply $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to an integration path that is a circle of radius r, where $r < R$. We need to calculate just the magnitude of E so we can take absolute values.

EXECUTE: $\left|\oint \vec{E} \cdot d\vec{l}\right| = E(2\pi r)$

$$\Phi_B = B\pi r^2, \quad \left|-\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$$

$$\left|\oint \vec{E} \cdot d\vec{l}\right| = \left|-\frac{d\Phi_B}{dt}\right| \text{ implies } E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|$$

$$E = \tfrac{1}{2} r \left|\frac{dB}{dt}\right|$$

$$B = \mu_0 nI, \text{ so } \frac{dB}{dt} = \mu_0 n \frac{dI}{dt}$$

Thus $E = \tfrac{1}{2} r \mu_0 n \dfrac{dI}{dt} = \tfrac{1}{2}(0.00500 \text{ m})(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(900 \text{ m}^{-1})(60.0 \text{ A/s}) = 1.70 \times 10^{-4} \text{ V/m}$

(b) $r = 0.0100$ cm is still inside the solenoid so the expression in part (a) applies.

$$E = \tfrac{1}{2} r \mu_0 n \frac{dI}{dt} = \tfrac{1}{2}(0.0100 \text{ m})(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(900 \text{ m}^{-1})(60.0 \text{ A/s}) = 3.39 \times 10^{-4} \text{ V/m}$$

EVALUATE: Inside the solenoid E is proportional to r, so E doubles when r doubles.

29.31. **IDENTIFY:** Apply Eq.(29.1) with $\Phi_B = \mu_0 niA$.

SET UP: $A = \pi r^2$, where $r = 0.0110\text{ m}$. In Eq.(29.11), $r = 0.0350\text{ m}$.

EXECUTE: $|\mathsf{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = \left|\dfrac{d}{dt}(BA)\right| = \left|\dfrac{d}{dt}(\mu_0 niA)\right| = \mu_0 nA\left|\dfrac{di}{dt}\right|$ and $|\mathsf{E}| = E(2\pi r)$. Therefore,

$\left|\dfrac{di}{dt}\right| = \dfrac{E2\pi r}{\mu_0 nA}$. $\left|\dfrac{di}{dt}\right| = \dfrac{(8.00\times10^{-6}\text{ V/m})2\pi(0.0350\text{ m})}{\mu_0(400\text{ m}^{-1})\pi(0.0110\text{ m})^2} = 9.21\text{ A/s}$.

EVALUATE: Outside the solenoid the induced electric field decreases with increasing distance from the axis of the solenoid.

29.32. **IDENTIFY:** A changing magnetic flux through a coil induces an emf in that coil, which means that an electric field is induced in the material of the coil.

SET UP: According to Faraday's law, the induced electric field obeys the equation $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$.

EXECUTE: **(a)** For the magnitude of the induced electric field, Faraday's law gives

$$E2\pi r = d(B\pi r^2)/dt = \pi r^2\, dB/dt$$

$$E = \frac{r}{2}\frac{dB}{dt} = \frac{0.0225\text{ m}}{2}(0.250\text{ T/s}) = 2.81\times10^{-3}\text{ V/m}$$

(b) The field points toward the south pole of the magnet and is decreasing, so the induced current is counterclockwise.

EVALUATE: This is a very small electric field compared to most others found in laboratory equipment.

29.7. Courant de déplacement et équations de Maxwell

- Le concept de courant de déplacement consiste à ajouter un terme dans la loi d'Ampère Eq. (28.20) pour la rendre semblable à l'Eq. (29.10).
- Ce terme supplémentaire est Eq. (29.14)
- Lire les pages 1013 et 1014
- Exemples :

29.34. **IDENTIFY:** Apply Eq.(29.14).

SET UP: $P = 3.5\times10^{-11}\text{ F/m}$

EXECUTE: $i_D = P\dfrac{d\Phi_E}{dt} = (3.5\times10^{-11}\text{ F/m})(24.0\times10^3\text{ V}\cdot\text{m/s}^3)t^2$. $i_D = 21\times10^{-6}\text{ A}$ gives $t = 5.0\text{ s}$.

EVALUATE: i_D depends on the rate at which Φ_E is changing.

29.35. **IDENTIFY:** Apply Eq.(29.14), where $P = KP_0$.

SET UP: $d\Phi_E/dt = 4(8.76\times10^3\text{ V}\cdot\text{m/s}^4)t^3$. $P_0 = 8.854\times10^{-12}\text{ F/m}$.

EXECUTE: $P = \dfrac{i_D}{(d\Phi_E/dt)} = \dfrac{12.9\times10^{-12}\text{ A}}{4(8.76\times10^3\text{ V}\cdot\text{m/s}^4)(26.1\times10^{-3}\text{ s})^3} = 2.07\times10^{-11}\text{ F/m}$. The dielectric constant is $K = \dfrac{P}{P_0} = 2.34$.

EVALUATE: The larger the dielectric constant, the larger is the displacement current for a given $d\Phi_E/dt$.

29.38. **IDENTIFY** and **SET UP:** Use $i_C = q/t$ to calculate the charge q that the current has carried to the plates in time t. The two equations preceeding Eq.(24.2) relate q to the electric field E and the potential difference between the plates. The displacement current density is defined by Eq.(29.16).

EXECUTE: **(a)** $i_C = 1.80 \times 10^{-3}\ \text{A}$

$q = 0$ at $t = 0$

The amount of charge brought to the plates by the charging current in time t is

$q = i_C t = (1.80 \times 10^{-3}\ \text{A})(0.500 \times 10^{-6}\ \text{s}) = 9.00 \times 10^{-10}\ \text{C}$

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} = \frac{9.00 \times 10^{-10}\ \text{C}}{(8.854 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00 \times 10^{-4}\ \text{m}^2)} = 2.03 \times 10^5\ \text{V/m}$$

$V = Ed = (2.03 \times 10^5\ \text{V/m})(2.00 \times 10^{-3}\ \text{m}) = 406\ \text{V}$

(b) $E = q/\epsilon_0 A$

$$\frac{dE}{dt} = \frac{dq/dt}{\epsilon_0 A} = \frac{i_C}{\epsilon_0 A} = \frac{1.80 \times 10^{-3}\ \text{A}}{(8.854 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00 \times 10^{-4}\ \text{m}^2)} = 4.07 \times 10^{11}\ \text{V/m}\cdot\text{s}$$

Since i_C is constant dE/dt does not vary in time.

(c) $j_D = \epsilon_0 \dfrac{dE}{dt}$ (Eq.(29.16)), with ϵ replaced by ϵ_0 since there is vacuum between the plates.)

$j_D = (8.854 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.07 \times 10^{11}\ \text{V/m}\cdot\text{s}) = 3.60\ \text{A/m}^2$

$i_D = j_D A = (3.60\ \text{A/m}^2)(5.00 \times 10^{-4}\ \text{m}^2) = 1.80 \times 10^{-3}\ \text{A};\ i_D = i_C$

EVALUATE: $i_C = i_D$. The constant conduction current means the charge q on the plates and the electric field between them both increase linearly with time and i_D is constant.

29.39. **IDENTIFY:** Ohm's law relates the current in the wire to the electric field in the wire. $j_D = \epsilon \dfrac{dE}{dt}$. Use Eq.(29.15) to calculate the magnetic fields.

SET UP: Ohm's law says $E = \rho J$. Apply Ohm's law to a circular path of radius r.

EXECUTE: **(a)** $E = \rho J = \dfrac{\rho I}{A} = \dfrac{(2.0 \times 10^{-8}\ \Omega\cdot\text{m})(16\ \text{A})}{2.1 \times 10^{-6}\ \text{m}^2} = 0.15\ \text{V/m}.$

(b) $\dfrac{dE}{dt} = \dfrac{d}{dt}\left(\dfrac{\rho I}{A}\right) = \dfrac{\rho}{A}\dfrac{dI}{dt} = \dfrac{2.0 \times 10^{-8}\ \Omega\cdot\text{m}}{2.1 \times 10^{-6}\ \text{m}^2}(4000\ \text{A/s}) = 38\ \text{V/m}\cdot\text{s}.$

(c) $j_D = \epsilon_0 \dfrac{dE}{dt} = \epsilon_0 (38\ \text{V/m}\cdot\text{s}) = 3.4 \times 10^{-10}\ \text{A/m}^2.$

(d) $i_D = j_D A = (3.4 \times 10^{-10}\ \text{A/m}^2)(2.1 \times 10^{-6}\ \text{m}^2) = 7.14 \times 10^{-16}\ \text{A}$. Eq.(29.15) applied to a circular path of radius r gives $B_D = \dfrac{\mu_0 I_D}{2\pi r} = \dfrac{\mu_0 (7.14 \times 10^{-16}\ \text{A})}{2\pi (0.060\ \text{m})} = 2.38 \times 10^{-21}\ \text{T}$, and this is a negligible contribution.

$$B_C = \frac{\mu_0 I_C}{2\pi r} = \frac{\mu_0}{2\pi}\frac{(16\ \text{A})}{(0.060\ \text{m})} = 5.33 \times 10^{-5}\ \text{T}.$$

EVALUATE: In this situation the displacement current is much less than the conduction current.

- Lire les pp. 1015 et 1016. Remarquer la symétrie dans les équations de Maxwell.