PHYS 230 Sections EB02/B02
Electricity and Magnetism
Final Exam
24 April 2007

MEC 4-1/4-3 9:00 AM – 12:00 PM Instructor: Dr. David Lawrie

NAME:____________________________________ **ID #** ______________________

A one-page formula sheet with student written notes is permitted.
No other notes are permitted, including notes stored electronically in a calculator.
Calculators without communications features are permitted.
All other electronic equipment prohibited.

Grading:

Multiple choice questions (Part A) grading is as follows:

2 or 3 choice questions:
 Worth 2 or 3 points for the correct answer. No partial credit
5 choice questions:
 5 points for the correct answer if no other answers selected
 3 points if 2 answers selected, one of which is correct.
 1 point if 3 answers selected, one of which is correct.
 0 points if the correct answer is not among the selections.

There is only one correct answer to each multiple choice question. If you believe the correct answer is not listed, ASK, then, if that does not help, choose the best (closest) answer.

Long answer questions (Part B) Answer ALL questions in the exam booklet.

Points available are indicated with each question. Partial credit is possible and intended – EXPLAIN your reasoning clearly (a logical series of short statements, formulas and diagrams (if needed) is all that is required for explanation. Don't waste time writing a lengthy essay. Do show all steps. Maximum points will be awarded for clear, concise and correct work. The correct numerical final answer is worth a small fraction of the total points.

There are 10 pages to this exam. The last 2 are a list of physical constants and a table of integrals and derivatives. It may be separated from the exam. Do not separate other pages.

IF ANYTHING IS UNCLEAR, PLEASE ASK!

Part A Multiple Choice (52 points, 36 % of total) CIRCLE answers directly on this test paper.

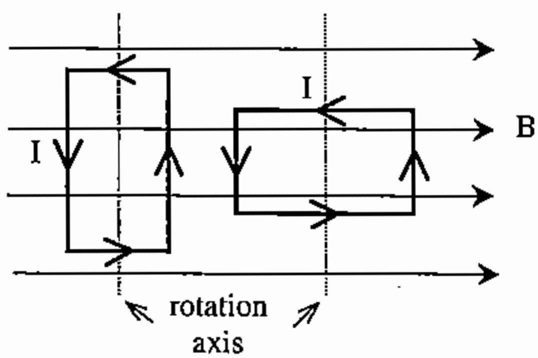
1. An electric dipole is composed of equal magnitude but opposite sign point charges separated by a distance d. At distances r >> d (r is the distance from the centre of the dipole to a point of interest), the electric field is:

   A. spherically symmetric and decreases as $1/r^3$.
   B. not spherically symmetric and decreases as $1/r^3$.
   C. not spherically symmetric and decreases as $1/r^2$.

2. A magnetic field, which does not change with time, exerts no force on a stationary charged particle.

   A. True.
   B. False.

3. A flat rectangular coil (dimensions L x 2L) of wire carries a current I and can rotate about a vertical axis in either of the two orientations shown (the axis passes through the centre of the coil in both cases). There is a uniform magnetic field B directed to the right and parallel to the plane of the coil.



   The torque for the two orientations:

   A. is greater for the coil on the left.
   B. is greater for the coil on the right.
   C. is non-zero and the same (magnitude and direction) for both.
   D. is non-zero, the same magnitude, but opposite in direction.
   E. is zero for both since the plane of the loop is parallel to the field.

4. A constant, uniform magnetic field acting on a charged particle moving perpendicular to the field does no work on the particle.

   A. True.
   B. False.

5. In SI units, is more energy stored in a unit of electric field (E = 1 V/m) or a unit of magnetic field (B = 1 T)? In other words, in vacuum, the ratio of magnetic field energy to electric field energy is:

A. $\left(\frac{\mu_o}{\varepsilon_o}\right)$, therefore more energy is stored in the magnetic field.

B. $\left(\frac{\varepsilon_o}{\mu_o}\right)$, therefore more energy is stored in the electric field.

C. $(\mu_o \varepsilon_o)$, therefore more energy is stored in the electric field.

D. $\left(\frac{1}{\mu_o \varepsilon_o}\right)$, therefore more energy is stored in the magnetic field.

E. 1, the same energy is stored per unit of electric or magnetic field.

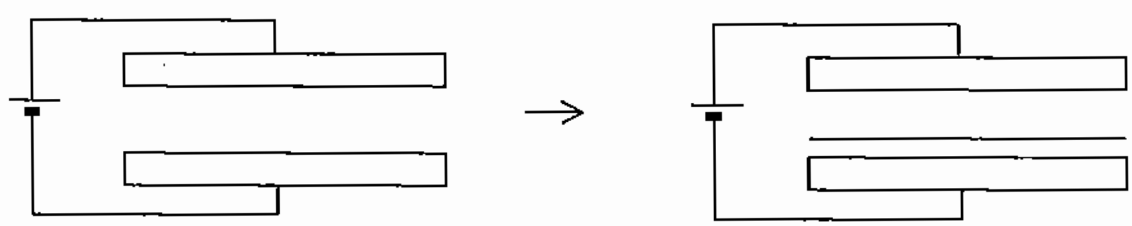
6. Given 3 identical capacitors, and using them all, how many different equivalent capacitances can be made?

A. 2
B. 4
C. 6

7. Gauss' law for magnetism states that the net magnetic flux through any surface is always zero.

A. True .
B. False.

8. An air filled parallel plate capacitor (Area A, plate separation d) is connected to a battery of voltage V. A very thin, initially uncharged metal plate of Area A is then (very carefully) inserted horizontally a distance d/4 from the bottom plate.



As a result,

A. The charge stored changes, but the voltage is constant.
B. The charge stored is constant, but the voltage changes.
C. Nothing changes

The next 5 questions refer to the following situation:

A hollow conducting sphere (inner radius 15.0 cm, outer radius 17.0 cm), has a surface charge density of +5.51 nC/m^2 on its outer surface. A solid conducting sphere (radius 2.00 cm) is located at the centre of the hollow sphere and has a net charge of – 2.00 nC.

9. The hollow sphere has a net charge of:

   A. + 4.00 nC
   B. + 2.00 nC
   C. zero.
   D. – 2.00 nC
   E. – 4.00 nC

10. The radial component of the electric field 8.00 cm from the centre of the spheres is closest to:

    A. + 2.81 kV/m.
    B. + 1.40 kV/m.
    C. zero.
    D. – 1.40 kV/m .
    E. – 2.81 kV/m.

11. The radial component of the electric field 16.00 cm from the centre of the spheres is closest to:

    A. + 1.40 kV/m.
    B. + 702 V/m.
    C. zero.
    D. – 702 V/m.
    E. – 1.40 kV/m.

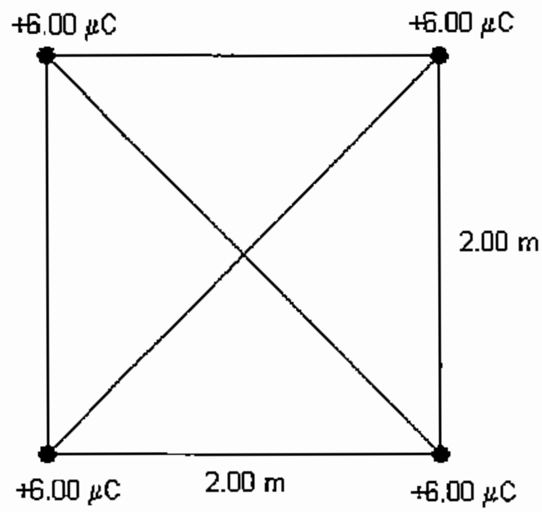
12. A grounded conducting wire is now touched to the outer surface of the hollow sphere and removed once the system has reached equilibrium. As a result, the charge density on the outer surface of the hollow sphere is now zero, but that on the inner surface remains unchanged.

    A. True.
    B. False.

13. Gauss' Law is the ONLY principle required to answer questions 9 to 12.

    A. True.
    B. False.

14. Four equal point charges (q = 6.00 μC) are arranged at the corners of a 2.00 m square as shown.



What are the magnitudes of the electric potential (V) and electric field (E) at the centre of the square?

A. V = 0 kV, E = 0 kV/m (Both cancel due to symmetry)
B. V = 38.1 kV, E = 0 kV/m
C. V = 153 kV, E = 0 kV/m
D. V = 153 kV, E = 108 kV/m
E. V = 0 kV, E = 108 kV/m

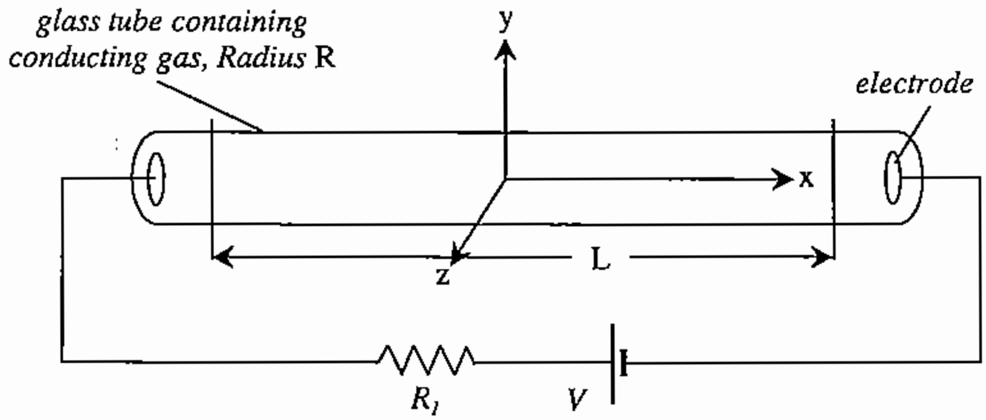
15. The earth's magnetic field is to be used as a generator to power a 1/2 watt flashlight bulb. The bulb has a resistance of 18 Ω. The generator consists of a coil of wire (1000 turns, 1.00 m in diameter.) that is oriented so that it can rotate about an axis perpendicular to the earth's magnetic field (5.00×10^{-5} T). How fast (rotational speed) must it be turned in order to light the bulb?

A. 1.22 revolutions/s
B. 12.2 revolutions/s
C. 122 revolutions/s

(End of Part A)

Part B Long Answer (92 points = 64% of total)

1. (24 Points) Non uniform current distributions in conductors actually do have practical applications. Electrical discharges in ionized gasses are one, for example in neon signs and fluorescents lights. The gas is enclosed in a glass tube with electrodes at either end, and these are connected to a power supply that provides a suitable voltage and current.



The current spreads out from the electrodes, but remains confined within the tube. For a tube that is much longer than its diameter, away from the electrodes ($-\frac{L}{2} \le x \le +\frac{L}{2}$), the current density will be radially symmetric, but more concentrated at the centre. As a crude model, consider a *conventional current* density given by:

$$J(r) = \begin{cases} J_o\left(1 - \frac{r}{R}\right) & r \le R \\ 0 & r \ge R \end{cases}$$

where $J_o = 955 \text{ A/m}^2$ is a constant and R = 5.00 cm.

a) (4 points) Use the given current density to find the total current flowing in the circuit.

b) (4 points) Use Ampere's law to find a formula for the magnetic field for distances r > R from the origin in the y-z plane. (Note that the tube is sufficiently long that end effects can be neglected.)

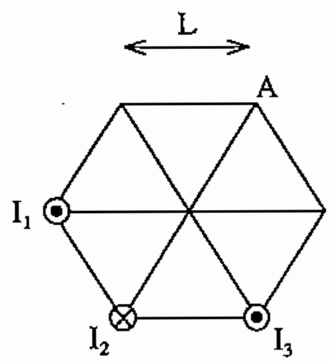
c) (8 points) Again, use Ampere's Law to find a formula for the magnetic field for distances r < R, in the y-z plane.

d) (2 points) What is the value of the magnetic field at the surface of the tube (r = R)?

e) (4 points) Make a sketch (graph) of the magnetic field magnitude as a function of r for all r.

f) (2 points) At what value of r does the magnetic field reach a maximum? What is the magnetic field magnitude at this point?

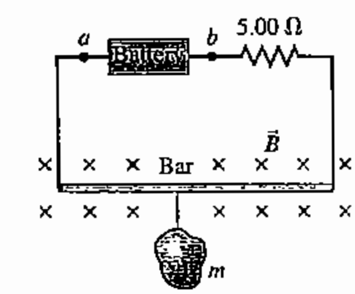
2. (20 points) Three long straight wires carry equal magnitude currents (I = 6.00 A) and are at the corners of a regular hexagon (edge length L = 15.0 cm) as shown. The wires are perpendicular to the plane of the page. I_1 and I_3 have carry current out of the page, and I_2 carries current into it. (The lines in the hexagon are for reference.)



a) (5 points) In the exam booklet, make a sketch of the magnetic field at point A due to each wire and indicate the direction of the net magnetic field at point A. Be sure to include a reference in your digaram that indicates the direction.

b) (15 points) Determine the net magnitude and direction of the magnetic field at A due to all three wires.

3. (13 points) The diagram shows a schematic of a magnetic balance. It consists of a counterweighted bar that is exactly balanced when the mass m is not attached, and no current flows through the bar. The 60. cm long bar is in a region of uniform magnetic field B = 0.75 T and is constrained to move in the vertical direction only. The resistance of the entire circuit is exactly 5 Ω. In order to weigh an object m, the battery voltage is adjusted so that the bar is again exactly balanced, when m is attached. At 12 volts, the battery voltage can be controlled to correct for imbalances of as little as 1 mV, i.e. V = 12.000 ± 0.001 volts. Given this voltage (assume all other constants mentioned are exact).



a) (3 points) Which terminal of the battery should be positive, given that gravity (g = 9.81 m/s^2) acts downwards? Why?

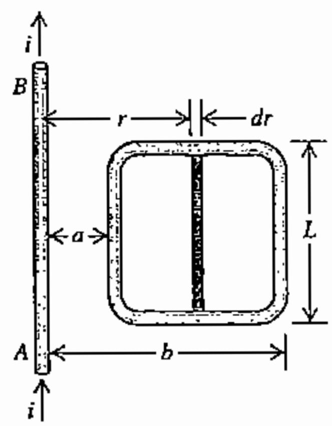
b) (5 points) What is the mass m for this voltage ?

c) (5 points) What is the uncertainty in the mass?

4. (15 points) A sample of sodium chloride is to be analyzed in a mass spectrometer. The sample is ionized, producing ions Na^+ (mass = 3.816×10^{-26} kg) and Cl^- (mass = 5.887×10^{-26} kg). The ions are passed through a velocity selector and leave with a speed of v = 8.1×10^4 m/s They then enter a region of constant magnetic field B = 0.400 T directed perpendicularly to their speed.

    a) (3 points) Determine magnitude and direction of the electric field in the velocity selector if the ions leave with speed v upwards and the magnetic field is of magnitude 0.02 T.
    b) (3 points) In the exam booklet, make a sketch of the motion of the ions when in the constant magnetic field region. Show an overhead view with the constant magnetic field directed out of the page and ions entering from the bottom and initially moving to upward. The ions move in 1/2 circle paths and reach detectors located at the edge of the constant field region. Clearly label the paths for the Na^+ and Cl^- ions.
    c) (9 points) Determine the distances the detectors need to be located from the injection point in order to intercept the ions.

5. (20 points) The figure shows a square conducting loop of wire near a long current carrying wire. At time t = 0, the current in the long wire is 1.2 A and is increasing at a constant rate of 9.6 A/s. The loop has total resistance R = 0.55 Ω, and edge length L = 24.0 cm. a = 12.0 cm and b = 36.0 cm.



    a) (2 points) Determine the direction of the induced current in the loop.
    b) (3 points) Is there a net force on the loop? If so, in which direction? If not, why not?
    c) (8 points) Determine a general formula for the magnetic flux through the loop. *(Hint integration)*
    d) (5 points) Determine the induced EMF in the loop.
    e) (2 points) Determine a numerical value for the induced current.

(End of Part B)

Potentially Useful Information:

This exam attempts to adhere to a 3 "sig-figs" rule = answers to any question should be reported to 3 significant digits. Needed data will be given (usually) to at least 3 "sig-figs". This means intermediate calculations should use at least 4 (nothing is really gained by using more though), and the constants below have way too many. Save time and round to 4 "sig-figs" if you choose to use any of the constants listed here.

Numerical Constants

*FUNDAMENTAL PHYSICAL CONSTANTS**

Name	*Symbol*	*Value*
Speed of light	c	2.99792458×10^8 m/s
Magnitude of charge of electron	e	$1.602176462(63) \times 10^{-19}$ C
Gravitational constant	G	$6.673(10) \times 10^{-11}$ N · m^2/kg^2
Planck's constant	h	$6.62606876(52) \times 10^{-34}$ J · s
Boltzmann constant	k	$1.3806503(24) \times 10^{-23}$ J/K
Avogadro's number	N_A	$6.02214199(47) \times 10^{23}$ molecules/mol
Gas constant	R	8.314472(15) J/mol · K
Mass of electron	m_e	$9.10938188(72) \times 10^{-31}$ kg
Mass of proton	m_p	$1.67262158(13) \times 10^{-27}$ kg
Mass of neutron	m_n	$1.67492716(13) \times 10^{-27}$ kg
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Wb/A · m
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854187817 \ldots \times 10^{-12}$ C^2/N · m^2
	$1/4\pi\epsilon_0$	$8.987551787 \ldots \times 10^9$ N · m^2/C^2

*OTHER USEFUL CONSTANTS**

Mechanical equivalent of heat		4.186 J/cal (15° calorie)
Standard atmospheric pressure	1 atm	1.01325×10^5 Pa
Absolute zero	0 K	−273.15°C
Electron volt	1 eV	$1.602176462(63) \times 10^{-19}$ J
Atomic mass unit	1 u	$1.66053873(13) \times 10^{-27}$ kg
Electron rest energy	$m_e c^2$	0.510998902(21) MeV
Volume of ideal gas (0°C and 1 atm)		22.413996(39) liter/mol
Acceleration due to gravity (standard)	g	9.80665 m/s^2

*Source: National Institute of Standards and Technology (http://physics.nist.gov/cuu). Numbers in parentheses show the uncertainty in the final digits of the main number; for example, the number 1.6454(21) means 1.6454 ± 0.0021. Values shown without uncertainties are exact.

DERIVATIVES AND INTEGRALS

In what follows, the letters u and v stand for any functions of x, and a and m are constants. To each of the indefinite integrals should be added an arbitrary constant of integration. The *Handbook of Chemistry and Physics* (CRC Press Inc.) gives a more extensive tabulation.

1. $\frac{dx}{dx} = 1$
2. $\frac{d}{dx}(au) = a\frac{du}{dx}$
3. $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$
4. $\frac{d}{dx}x^m = mx^{m-1}$
5. $\frac{d}{dx}\ln x = \frac{1}{x}$
6. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
7. $\frac{d}{dx}e^x = e^x$
8. $\frac{d}{dx}\sin x = \cos x$
9. $\frac{d}{dx}\cos x = -\sin x$
10. $\frac{d}{dx}\tan x = \sec^2 x$
11. $\frac{d}{dx}\cot x = -\csc^2 x$
12. $\frac{d}{dx}\sec x = \tan x \sec x$
13. $\frac{d}{dx}\csc x = -\cot x \csc x$
14. $\frac{d}{dx}e^u = e^u\frac{du}{dx}$
15. $\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$
16. $\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$

1. $\int dx = x$
2. $\int au\,dx = a\int u\,dx$
3. $\int (u + v)\,dx = \int u\,dx + \int v\,dx$
4. $\int x^m\,dx = \frac{x^{m+1}}{m+1} \quad (m \neq -1)$
5. $\int \frac{dx}{x} = \ln |x|$
6. $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
7. $\int e^x\,dx = e^x$
8. $\int \sin x\,dx = -\cos x$
9. $\int \cos x\,dx = \sin x$
10. $\int \tan x\,dx = \ln |\sec x|$
11. $\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$
12. $\int e^{-ax}\,dx = -\frac{1}{a}e^{-ax}$
13. $\int xe^{-ax}\,dx = -\frac{1}{a^2}(ax + 1)e^{-ax}$
14. $\int x^2e^{-ax}\,dx = -\frac{1}{a^3}(a^2x^2 + 2ax + 2)e^{-ax}$
15. $\int_0^\infty x^ne^{-ax}\,dx = \frac{n!}{a^{n+1}}$
16. $\int_0^\infty x^{2n}e^{-ax^2}\,dx = \frac{1\cdot3\cdot5\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}$
17. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$
18. $\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{(x^2 + a^2)^{1/2}}$
19. $\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$

also: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)$