PHYS 230 Sections EB02/B02
Electricity and Magnetism
MidTerm Exam
15 February 2007

CEB 325 9:30 - 10:50 AM Instructor: Dr. David Lawrie

NAME:______________________________ **ID #** ____________________

A one-page formula sheet with student written notes is permitted.
No other notes are permitted, including notes stored electronically in a calculator.
Calculators without communications features are permitted.
All other electronic equipment prohibited.

Grading:

Multiple choice questions (Part A) grading is as follows:

- 2 or 3 choice questions:
  - Worth 2 or 3 points for the correct answer. No partial credit
- 5 choice questions:
  - 5 points for the correct answer if no other answers selected
  - 3 points if 2 answers selected, one of which is correct.
  - 1 point if 3 answers selected, one of which is correct.
  - 0 points if the correct answer is not among the selections.

There is only one correct answer to each multiple choice question. If you believe the correct answer is not listed, ASK, then, if that does not help, choose the best (closest) answer.

Long answer questions (Part B)

Points available are indicated with each question. Partial credit is possible and intended – EXPLAIN your reasoning clearly (a logical series of short statements, formulas and diagrams (if needed) is all that is required for explanation. Don't waste time writing a lengthy essay. Do show all steps. Maximum points will be awarded for clear, concise and correct work. The correct numerical final answer is worth a small fraction of the total points.

There are 7 pages to this exam, the last 2 are a list of physical constants and a table of integrals and derivatives.

IF ANYTHING IS UNCLEAR, PLEASE ASK!

Part A Multiple Choice (40 points = 40% of total)

1. Two identically sized conducting spheres have initial charges of q_1 and q_2. They are touched together and then separated. After touching and separating, each will have the same charge, q, of:

   A. $q = \frac{q_1 + q_2}{2}$
   B. $q = \frac{q_1 - q_2}{2}$

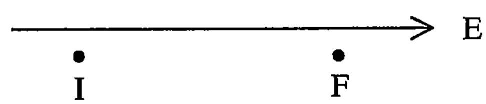
2. Coulomb's law can be used to calculate the force on a test charge for non-symmetric charge distributions.

   A. True
   B. False

3. The electric field near the Earth's surface is (on average) about 150 V/m, directed downwards (towards the Earth's surface). A negatively charged particle of mass M = 80.0 mg, and charge q = -5.23 μC is released from rest near the Earth's surface. After 1.00 second, it will have moved a distance of:
   (Only gravity and the electric force are relevant, all other forces (air resistance, etc) may be neglected)

   A. 0.00 m
   B. 4.91 m
   C. 9.81 m

4. An electron moves in the direction of a uniform electric field from an initial position I to a final position F. As a result,



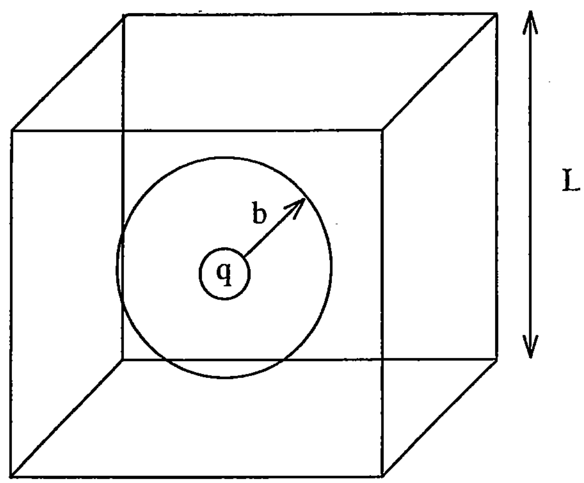
   A. its speed increases.
   B. its speed decreases.
   C. its potential energy remains constant.

5. Two identical point charges are separated by a distance d. Choose the most accurate statement:

   A. The electric field is zero while the electric potential is non-zero at d/2 (mid way between the charges).
   B. The electric field is non-zero while the electric potential is zero at d/2 (mid way between the charges).
   C. Both the electric field and the electric potential are zero at d/2.

The next 3 questions refer to the following situation:

An insulating sphere (radius a = 1.25 cm) has a charge q = + 6.00 μC uniformly distributed through out its volume. It is located at the centre of an otherwise empty spherical cavity (radius b = 5.00 cm) inside an electrically neutral metal (conducting) cube of edge length L = 25.0 cm. The centres of the sphere, cavity and cube coincide.



6. The surface charge density, σ_b, on the interior of the cavity is:

   A. $\sigma_b = -3.06 \times 10^{-3}$ C/m².
   B. $\sigma_b = -1.91 \times 10^{-4}$ C/m².
   C. $\sigma_b = +1.91 \times 10^{-4}$ C/m².
   D. $\sigma_b = +3.06 \times 10^{-3}$ C/m².
   E. Zero, since the cube is a conductor and all charge will be on the outer surface.

7. The surface charge density, σ_e, on the exterior surface of the cube is:

   A. uniform and $\sigma_e = +\frac{q}{6L^2}$.
   B. uniform and $\sigma_e = -\frac{q}{6L^2}$.
   C. non-uniform, positive and greatest at the corners.
   D. non-uniform, negative and greatest at the corners.
   E. non-uniform, positive and greatest at the centre of the cube faces.

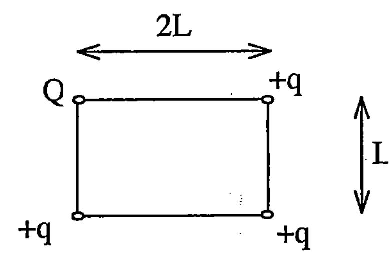
8. Every point that is physically part of the cube is at the same potential, including points inside its surface.

   A. True
   B. False

9. In the semi-classical Bohr model of the hydrogen atom, an electron is separated from a proton by the Bohr radius, $a_o = 5.29 \times 10^{-11}$ m. Determine the force exerted on the electron by the proton at this separation.

   A. 4.36×10^{-18} N
   B. 8.24×10^{-8} N
   C. 1.65×10^{-9} N
   D. 8.24×10^{8} N
   E. 5.15×10^{11} N

10. Four point charges are located at the corners of a rectangle as shown. What value should Q have if the electric potential is to be zero at the centre of the rectangle?



   A. $Q = +3\,q$
   B. $Q = +\sqrt{3}\,q$
   C. $Q = +\,q$
   D. $Q = -\sqrt{5}\,q$
   E. $Q = -3\,q$

11. Two identical small conducting spheres are separated by 0.600 m. The spheres carry different amounts of charge and each sphere experiences an attractive force of 10.8 N. The total (net) charge of the spheres is –24 μC. The charge of the positive sphere is closest to:

   A. 12 μC
   B. 18 μC
   C. 24 μC
   D. 30 μC
   E. 36 μC

(End of Part A)

Part B Long Answer (60 points = 60% of total)

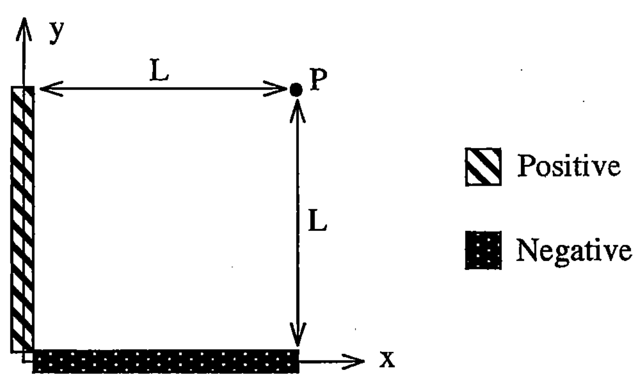
1. (34 points) Consider an insulating sphere of radius R with a spherically symmetric, but non-uniform positive charge density given by:

$$\rho(r) = \frac{C}{r} \quad r \le R$$
$$= 0 \quad r > R$$

C is a positive constant and the distribution has a total charge Q.

a) (5) Determine the constant C in terms of R, Q and other relevant constants.
b) (2) Use Gauss' Law to determine the electric field for $r > R$.
c) (5) Use Gauss' Law to determine the electric field for $r \le R$.
d) (3) Make a sketch (rough graph) of the electric field for all values of r.
e) (2) Determine the electric potential for points $r > R$, assuming a potential of zero at $r = \infty$.
f) (5) Determine the electric potential for points $r \le R$, assuming a potential of zero at $r = \infty$.
g) (3) Make a sketch of the potential for all r.
h) (2) Where (at what value(s) of r) is the electric field a minimum?
i) (2) Where (at what value(s) of r) is the electric potential a minimum?

2. (26 points) Two uniformly charged thin insulating rods are located on the x and y axes as shown.Each has a length L, total charge q, but have opposite signs for their charge.



a) (3) Based on symmetry considerations, estimate the direction of the net electric field at point P. (No calculations needed, but explain your reasoning clearly).
b) (10) Determine the components of the electric field at P produced by the positive rod alone.
c) (10) Determine the components of the electric field at P produced by the negative rod alone.
d) (3) Determine the magnitude and direction of the net electric field at P.

(End of Part B)

Potentially Useful Information:

This exam attempts to adhere to a 3 "sig-figs" rule = answers to any question should be reported to 3 significant digits. Needed data will be given (usually) to at least 3 "sig-figs". This means intermediate calculations should use at least 4 (nothing is really gained by using more though), and the constants below have way too many. Save time and round to 4 "sig-figs" if you choose to use any of the constants listed here.

Numerical Constants

*FUNDAMENTAL PHYSICAL CONSTANTS**

Name	*Symbol*	*Value*
Speed of light	c	2.99792458×10^{8} m/s
Magnitude of charge of electron	e	$1.602176462(63) \times 10^{-19}$ C
Gravitational constant	G	$6.673(10) \times 10^{-11}$ N · m²/kg²
Planck's constant	h	$6.62606876(52) \times 10^{-34}$ J · s
Boltzmann constant	k	$1.3806503(24) \times 10^{-23}$ J/K
Avogadro's number	N_A	$6.02214199(47) \times 10^{23}$ molecules/mol
Gas constant	R	8.314472(15) J/mol · K
Mass of electron	m_e	$9.10938188(72) \times 10^{-31}$ kg
Mass of proton	m_p	$1.67262158(13) \times 10^{-27}$ kg
Mass of neutron	m_n	$1.67492716(13) \times 10^{-27}$ kg
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Wb/A · m
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854187817 \ldots \times 10^{-12}$ $C^2/N \cdot m^2$
	$1/4\pi\epsilon_0$	$8.987551787 \ldots \times 10^{9}$ $N \cdot m^2/C^2$

*OTHER USEFUL CONSTANTS**

Mechanical equivalent of heat		4.186 J/cal (15° calorie)
Standard atmospheric pressure	1 atm	1.01325×10^{5} Pa
Absolute zero	0 K	−273.15°C
Electron volt	1 eV	$1.602176462(63) \times 10^{-19}$ J
Atomic mass unit	1 u	$1.66053873(13) \times 10^{-27}$ kg
Electron rest energy	$m_e c^2$	0.510998902(21) MeV
Volume of ideal gas (0°C and 1 atm)		22.413996(39) liter/mol
Acceleration due to gravity (standard)	g	9.80665 m/s²

*Source: National Institute of Standards and Technology (**http://physics.nist.gov/cuu**). Numbers in parentheses show the uncertainty in the final digits of the main number; for example, the number 1.6454(21) means 1.6454 ± 0.0021. Values shown without uncertainties are exact.

DERIVATIVES AND INTEGRALS

In what follows, the letters u and v stand for any functions of x, and a and m are constants. To each of the indefinite integrals should be added an arbitrary constant of integration. The *Handbook of Chemistry and Physics* (CRC Press Inc.) gives a more extensive tabulation.

1. $\frac{dx}{dx} = 1$
2. $\frac{d}{dx}(au) = a\frac{du}{dx}$
3. $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
4. $\frac{d}{dx}x^m = mx^{m-1}$
5. $\frac{d}{dx}\ln x = \frac{1}{x}$
6. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
7. $\frac{d}{dx}e^x = e^x$
8. $\frac{d}{dx}\sin x = \cos x$
9. $\frac{d}{dx}\cos x = -\sin x$
10. $\frac{d}{dx}\tan x = \sec^2 x$
11. $\frac{d}{dx}\cot x = -\csc^2 x$
12. $\frac{d}{dx}\sec x = \tan x \sec x$
13. $\frac{d}{dx}\csc x = -\cot x \csc x$
14. $\frac{d}{dx}e^u = e^u\frac{du}{dx}$
15. $\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$
16. $\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$

1. $\int dx = x$
2. $\int au\,dx = a\int u\,dx$
3. $\int (u+v)\,dx = \int u\,dx + \int v\,dx$
4. $\int x^m\,dx = \frac{x^{m+1}}{m+1} \quad (m \neq -1)$
5. $\int \frac{dx}{x} = \ln|x|$
6. $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
7. $\int e^x\,dx = e^x$
8. $\int \sin x\,dx = -\cos x$
9. $\int \cos x\,dx = \sin x$
10. $\int \tan x\,dx = \ln|\sec x|$
11. $\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$
12. $\int e^{-ax}\,dx = -\frac{1}{a}e^{-ax}$
13. $\int xe^{-ax}\,dx = -\frac{1}{a^2}(ax+1)e^{-ax}$
14. $\int x^2e^{-ax}\,dx = -\frac{1}{a^3}(a^2x^2 + 2ax + 2)e^{-ax}$
15. $\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$
16. $\int_0^\infty x^{2n}e^{-ax^2}\,dx = \frac{1\cdot3\cdot5\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}$
17. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2})$
18. $\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{(x^2+a^2)^{1/2}}$
19. $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$

also: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)$