PART 7

Discussion on different transforms

Mauricio Sacchi
University of Alberta, Edmonton, AB, Canada

Gabor representation

Data are represented by a superposition of modulated localized signals (Gaussian windows)

Fourier Synthesis

$$d(n) = \sum_k c_k e^{i2\pi kn/K}.$$

Non-local versus local Fourier Representations

Gabor Synthesis

$$d(n) = \sum_m \sum_k c_{mk} g(n - mL) e^{i2\pi kn/K}.$$

$$k_{rad} = 2\pi k/K, k = 0 \cdots K-1$$

Gabor representation

- Unlike Fourier basis functions, Gabor bases can model signals with spatially variant dips.
- Gabor reconstruction does not require small data patches
- Gabor bases are labeled by local wavenumbers and therefore, local dip.

$$d(n) = \sum_m \sum_k c_{mk} g(n - mL) e^{i2\pi kn/K}.$$

$m \rightarrow$ spatial center of window

$k \rightarrow$ wavenumber at m $\rightarrow$ local wavenumber

$$p \rightarrow \text{local dip} = \frac{\text{local wavenumber}}{\text{temporal frequency}}$$

Solving the problem

Expansion:

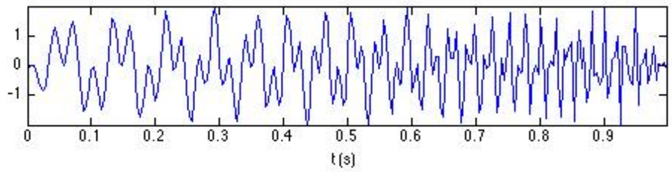
$$\vec{d} = G\vec{c} + \vec{n},$$

Minimize the cost:

$$J = \| S(\vec{d} - G\vec{c}) \|^2 + \mu^2 R(\vec{c})$$

Compressive norm (l1 or Cauchy or MWNI) to find sparse solutions

Superposition of two hyperbolic chirps

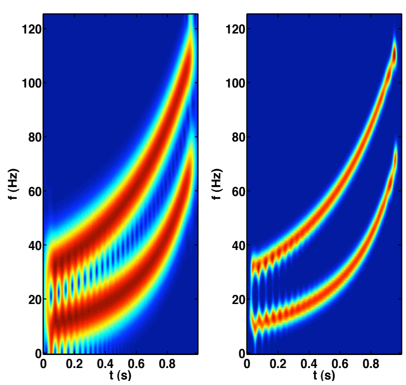


Each chirp with the following form: $s(t) = \cos(\frac{a}{b-t})$

$$e = (f_{\min} / f_{\max})^{1/2}$$
$$b = t_{\max} / (1-e)$$
$$a = 2\pi b^2$$

Superposition of two hyperbolic chirps

$|c_{mk}|$= Gabor spectrum

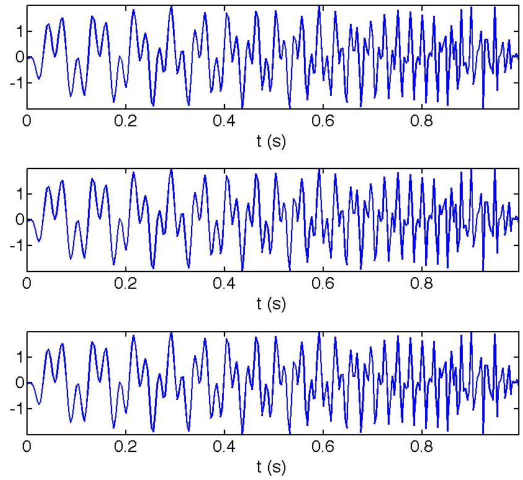


Minimum Norm=Gabor transform

Gabor coefficients with sparse (l_1) Inverted

Superposition of two hyperbolic chirps

Gabor expansion can recover the data

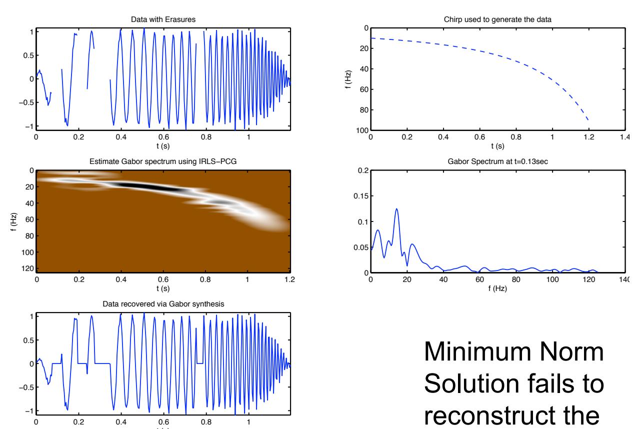


Original

Min Norm

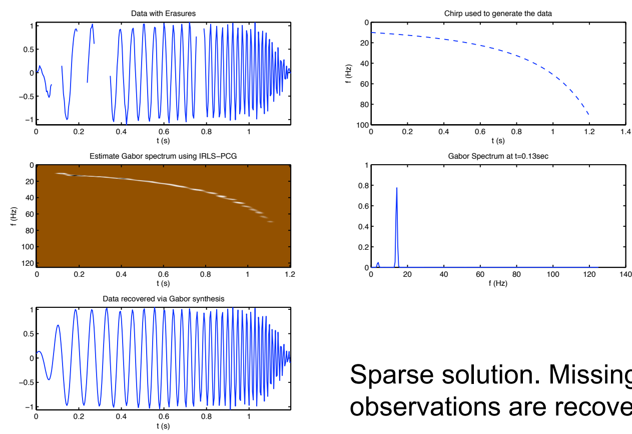
Sparse/Min Entropy sol.

Reconstruction of a chirp with erasures



Minimum Norm Solution fails to reconstruct the erasures

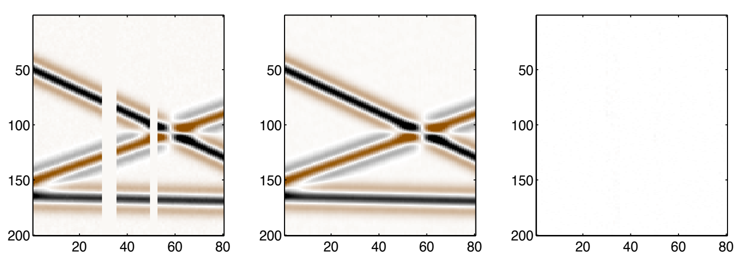
Reconstruction of a chirp with erasures



Sparse solution. Missing observations are recovered.

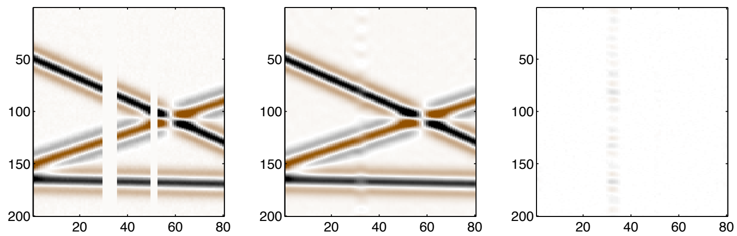
Why now works?. We are recovering the spectrum we should have obtained if the complete data were available. This works because the ideal representation should be sparse.

Reconstruction in the FX domain Spatially invariant dips (Fourier will work)



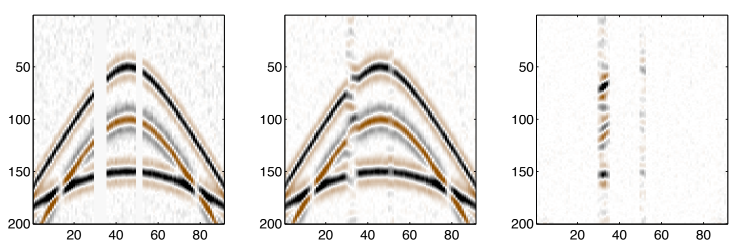
Fourier Reconstruction with Sparsity Constraint. Basis functions are complex exponentials spanning the complete aperture

Reconstruction in the FX domain
Spatially invariant dips (Gabor will work too)



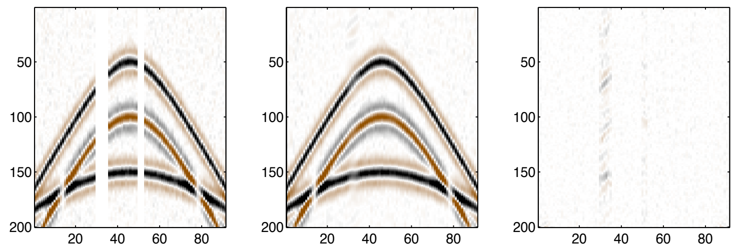
Gabor Reconstruction with Sparsity Constraint. Basis functions are complex Gabor atoms

Reconstruction in the FX domain
Dips are varying with space (Fourier will not work)



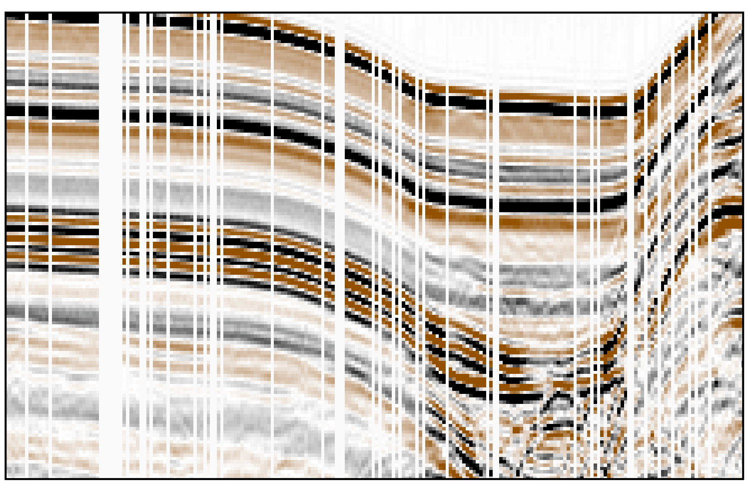
Fourier Reconstruction with Sparsity Constraint. Basis functions are complex exponentials spanning the complete aperture. Bad selection of basis functions. Therefore, the sparsity trick will make things even worse.

Reconstruction in the FX domain Dips are varying with space (Gabor will work)



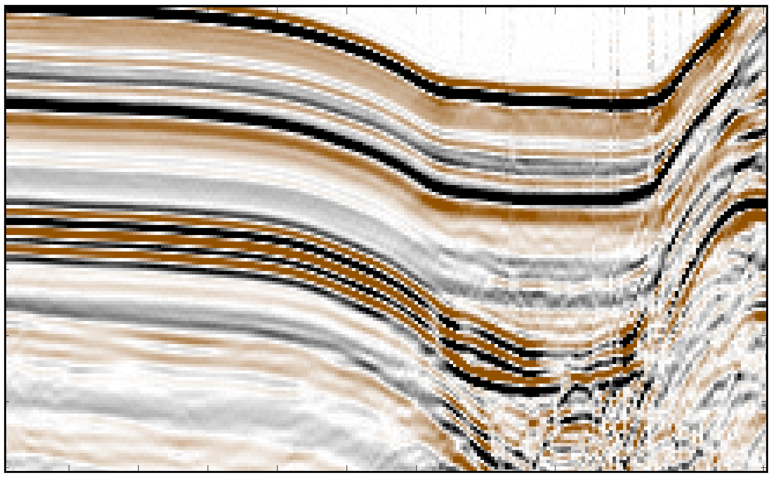
Gabor Reconstruction with Sparsity Constraint. Basis functions are complex Gabor atoms, local functions.

GOM data - input



GOM data - reconstruction

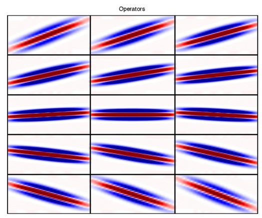
+- 50% amplitude clip



Methods based local Radon transforms

$$B(\omega, x, p) = h(x) S(\omega)\, e^{-i\omega x p}, \qquad a < x < b$$

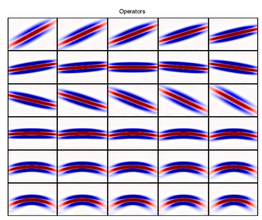
$$b(t, x, p) = \mathrm{F}^{-1}[B(\omega, x, p)]$$



Methods based local Radon transforms

$$B(\omega,x,q) = h(x)S(\omega)\, e^{-i\omega x^2 q}, \quad a < x < b$$

$$b(t,x,q) = \mathrm{F}^{-1}[B(\omega,x,q)]$$



Methods based local Radon transforms

One operator shifted in t-x and scaled

$$f(t',x')\, b(t-t',x-x',p)$$

Superposition of many operators

$$\sum_p \sum_{t'} \sum_{x'} f(t',x',p)\, b(t-t',x-x',p)$$

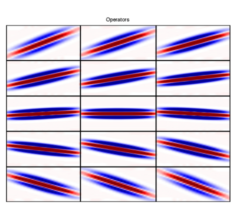
Methods based local Radon transforms

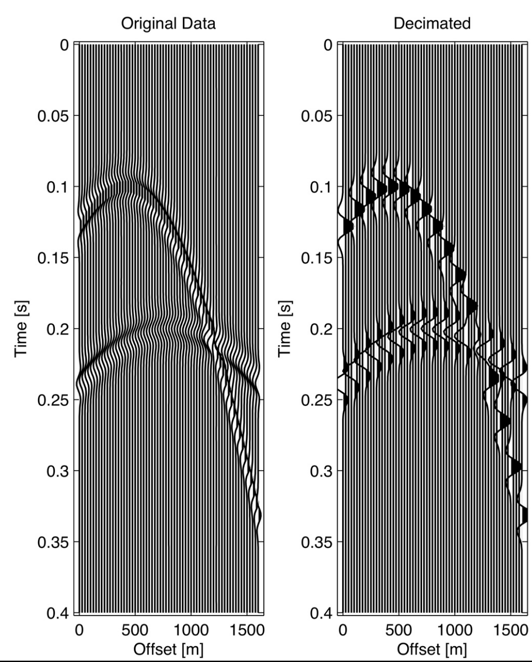
$$d(t,x) = \sum_k \sum_{x'} \sum_{t'} f(t',x',p_k)\, b(t-t',x-x',p_k)$$

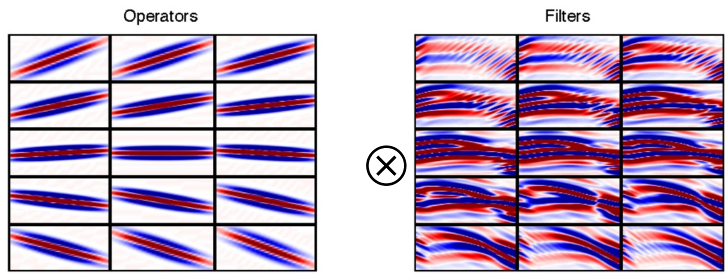
$$D = \sum_k B_k \otimes F_k$$
$$= \sum_k D_k$$

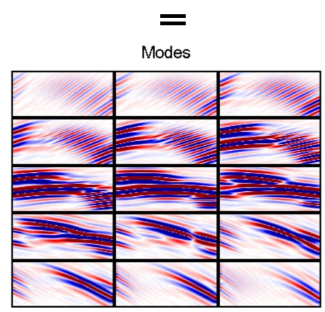
$$D_k = B_k * F_k$$

We will try to shape local operators into data…

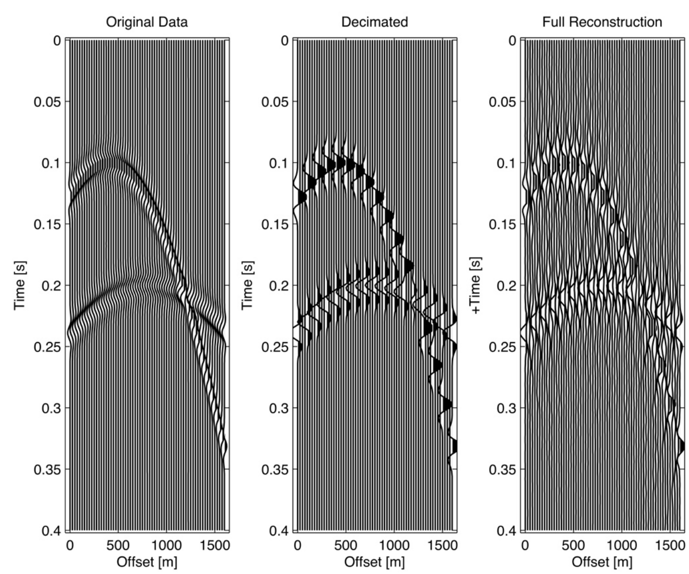




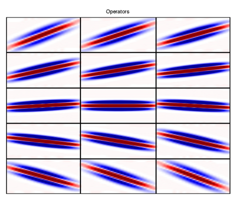


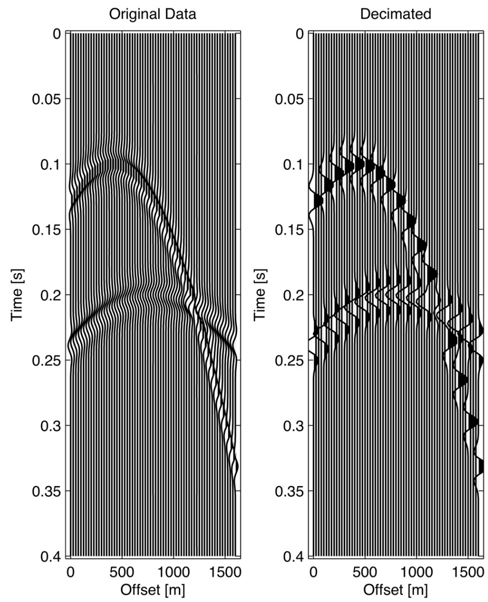


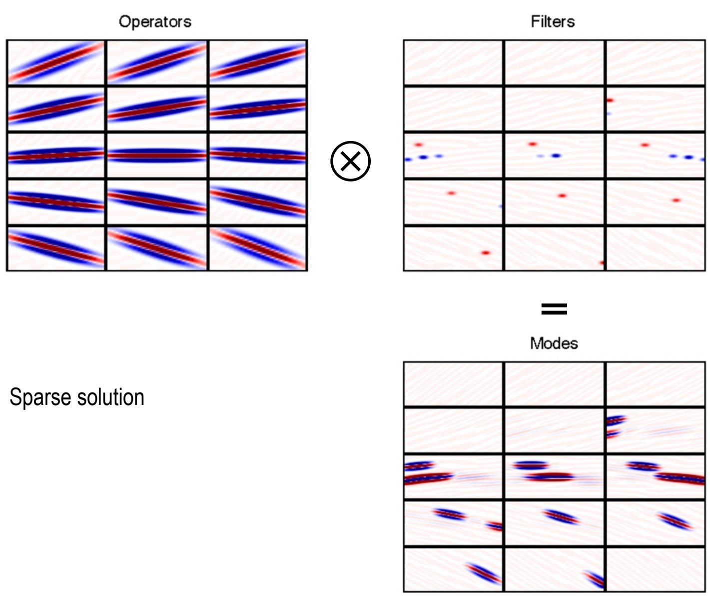
Minimum norm solution



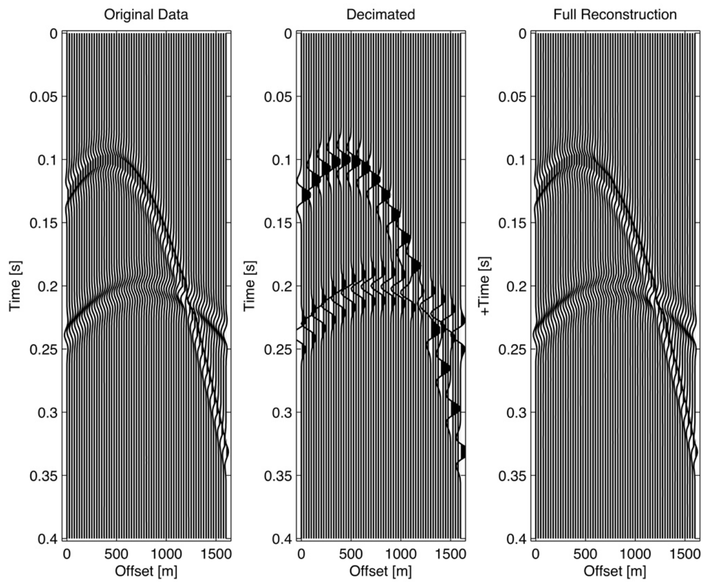
Minimum norm solution

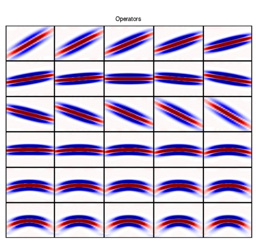




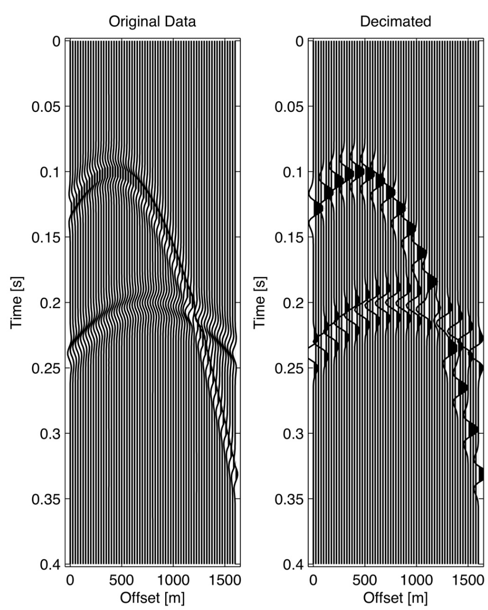


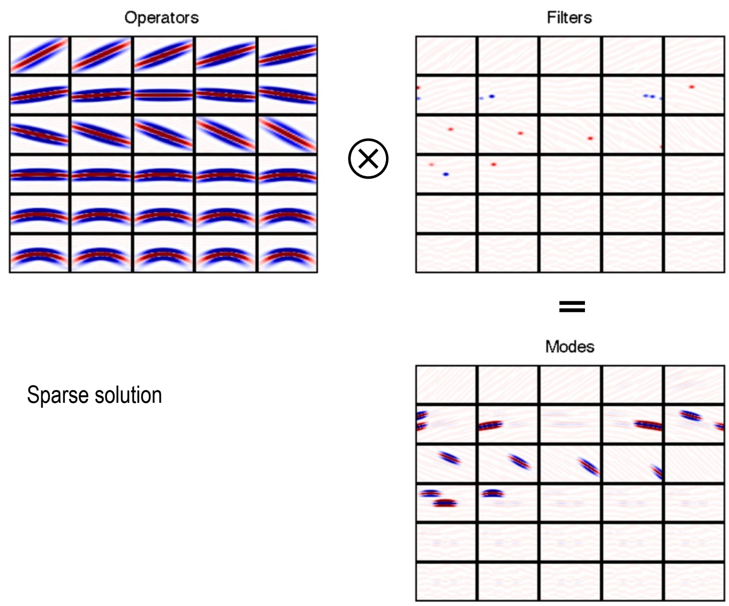
Sparse solution





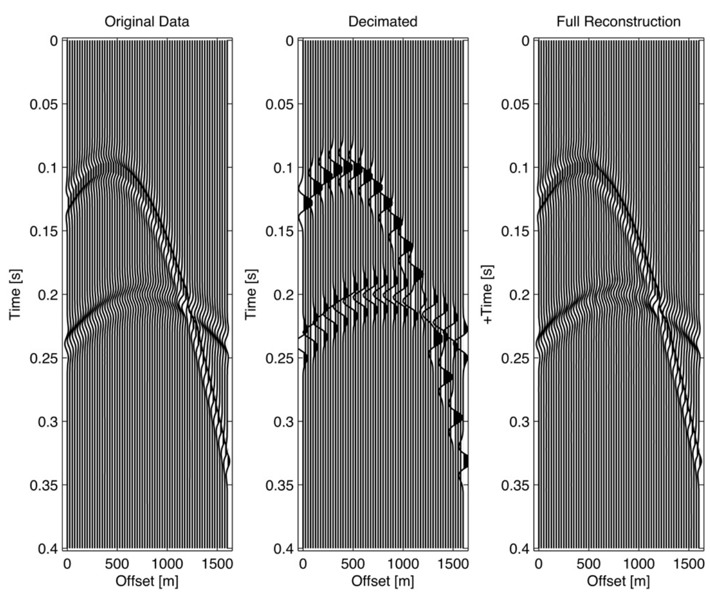
Minimum norm solution





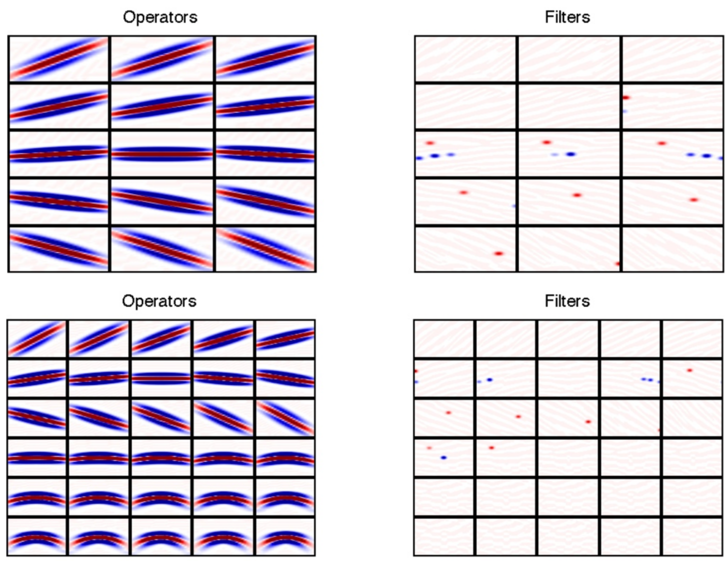
Sparse solution

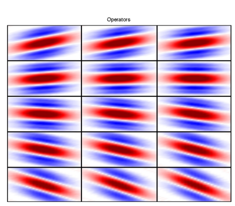
Part 7 - Other transforms 27

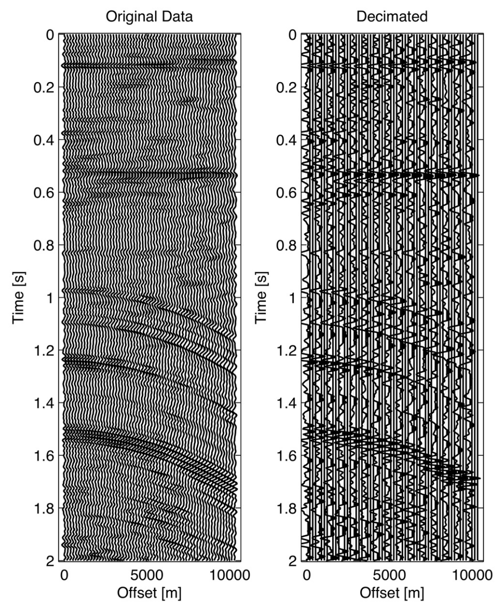


Sparse solution

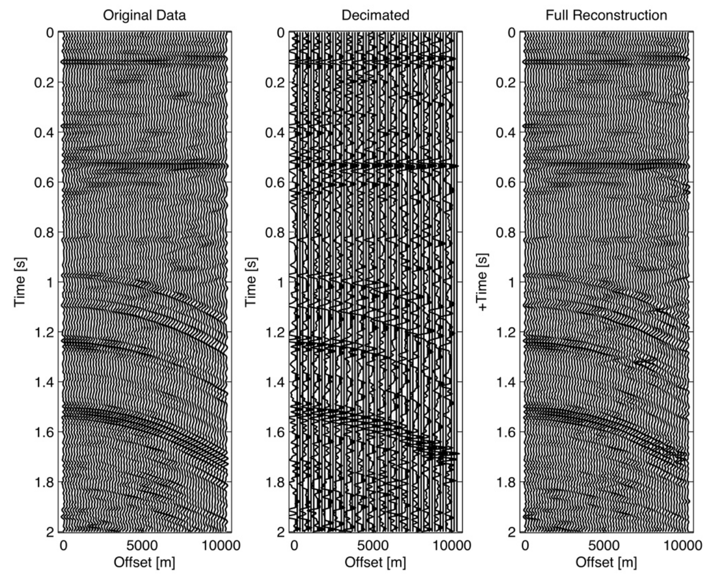
Part 7 - Other transforms 28







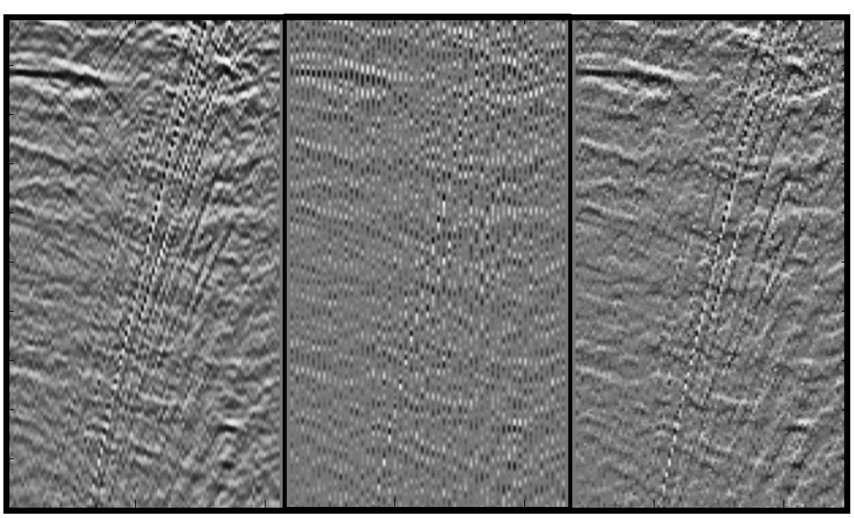
Remove 3, keep 1



Sparse solution

Part 7 - Other transforms 31

GOM reconstruction of an aliased data window



Part 7 - Other transforms 32

Curvelet representation

$$d(t,x) = \sum_{p,s,t',x'} w(t-t', x-x', p, s)\, c(t', x', p, s)$$

t', x': location

p: dip

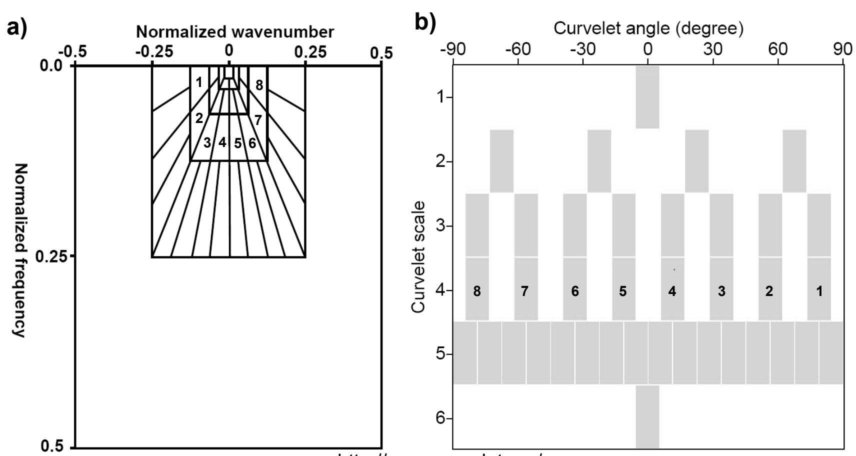
s: scale

Find coefficients by minimizing

$$J = \| S(\vec{d} - G\vec{c}) \|^2 + \mu^2 R(\vec{c})$$

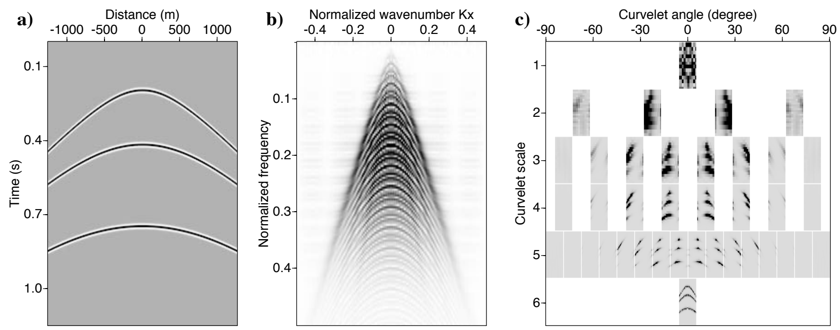
Curvelet representation

E. J. Candes, L. Demanet, D. L. Donoho, L. Yin, 2005



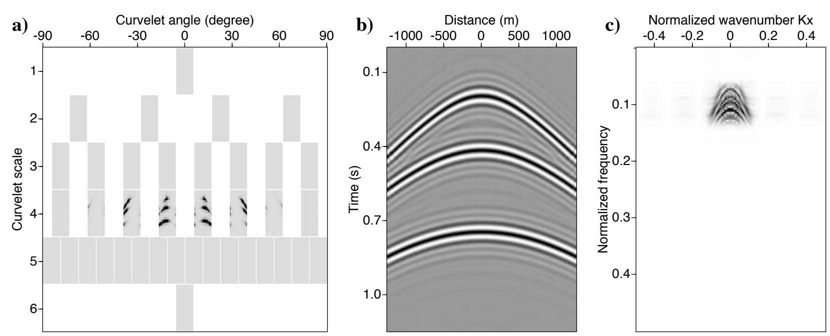
http://www.curvelet.org/

Curvelet representation



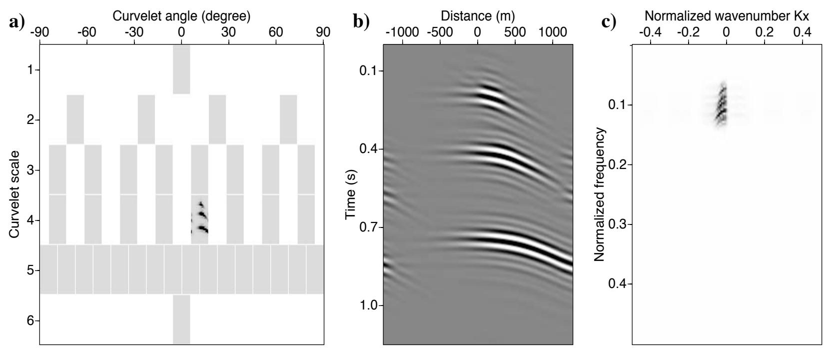
Naghizadeh's representation is described in the Recorder abstract. It basically attempts to give physical meaning to the coefficients. Each patch represents data at a given scale and directions

Data synthesis using all directions at one single scale

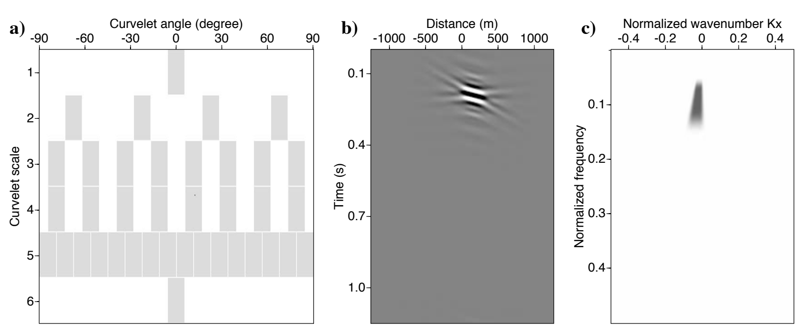


Angle can be converted to dip. *tan*(angle)=non-dimensional dip

Data synthesis using one directions and one scale



Data synthesis using one coefficient



Beyond alias curvelet interpolation

Find coefficients by minimizing

$$J = \| S(\vec{d} - G W_{p,s} \vec{c}) \|^2 + \mu^2 R(\vec{c})$$

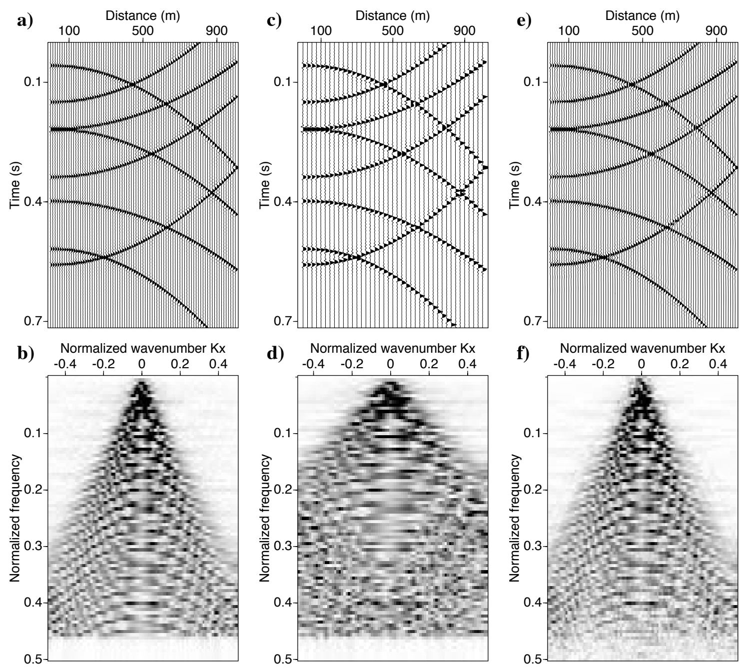
$W_{p,s}$: dip/scale dependent mask

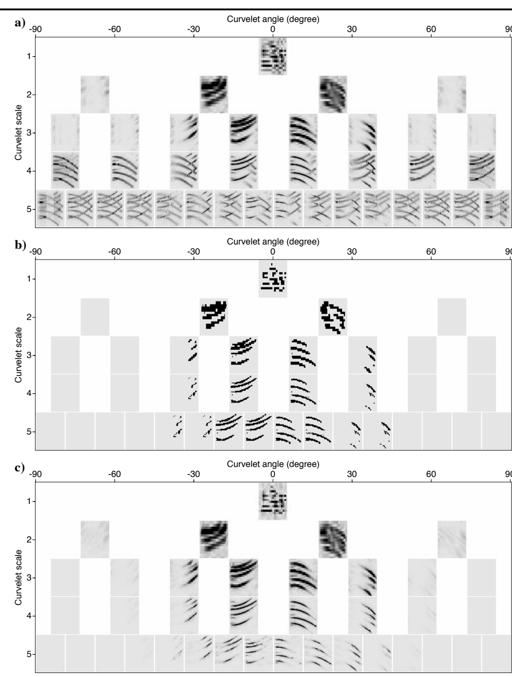
- Two passes process:
  - Identify alias-free coefficients from coarse scales
  - Define a mask (all pass function) for alias-free coefficients
  - Upscale mask
  - Solve the equation above
  - Reconstruct data via curvelet synthesis.

Naghizadeh, M. and M. Sacchi (2009). Beyond alias hierarchical scale curvelet interpolation of regularly and irregularly sampled seismic data, Geophysics 2011 (Special Issue on Reconstruction)

Beyond alias curvelet interpolation

- Spitz Prediction filter interpolation (1991)
  - PEF extracted from non-aliased low frequencies drive the reconstruction of aliased high frequency spatial data
- Curvelet beyond alias interpolation
  - Coarse scale coefficients drive the inversion of aliased high frequency components
- This is a nice example of classical / proven methods (Spitz 1991) driving the development of new strategies for data processing

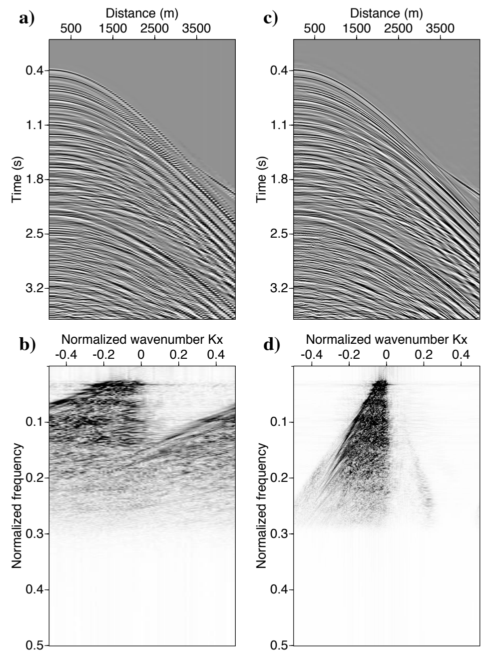


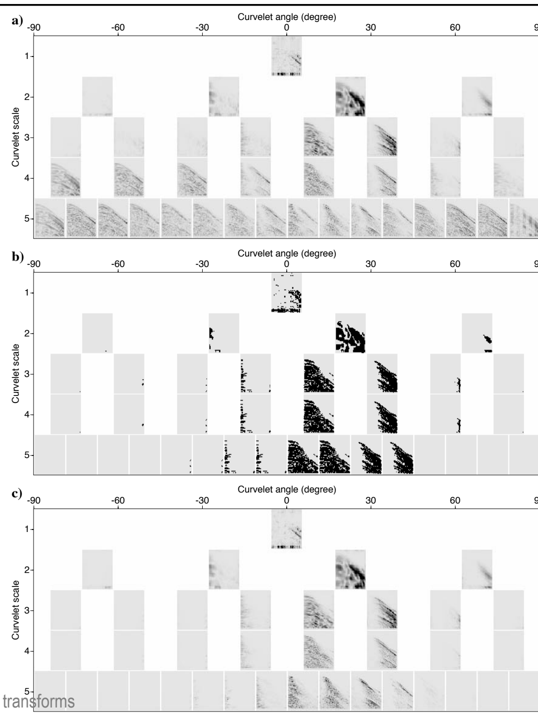


Raw coefficients. Scales 4 and 5 show alias

Mask used to constraint inversion

Final de-aliased coefficients computed via inversion





Raw coefficients. Scales 4 and 5 show alias

Mask used to constraint inversion

Final de-aliased coefficients computed via inversion

- Curvelet de-aliasing is similar to Spitz FX interpolation
  - Spitz 91, uses low frequency PEF to interpolated high frequency aliased data
  - Curverlet interpolation uses low scales to define regions of support of non-aliased curvelet coefficients at higher scales
  - Curvelet works well with random sampled data but all methods work well when the data are randomly sampled because there is no alias.
  - Good interpolators show handled cases when the input data is aliased.