Adding noise with a desired signal-to-noise ratio

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There is often confusion between different definitions of signal-to-noise ratio SNR. In order to communicate your results, I suggest to be clear and adopt one definition. For instance,

$$\text{SNR} = \frac{\text{Power of the clean signal}}{\text{Power of the additive noise}} = \frac{P_s}{P_n}$$

The observed signal is $d_k, k = 1 \dots N$. The signal can be written as a vector of length N that we call $\mathbf{d}$. The clean signal is given by $s_k, k = 1 \dots N$ and it can also be expressed as a vector of length N denoted by $\mathbf{s}$. Our task is to add noise to the data

$$d_k = s_k + \alpha\, n_k\,,$$

or

$$\mathbf{d} = \mathbf{s} + \alpha \mathbf{n}$$

where α is a scalar used to yield a predefined SNR. We define the power of the signal and noise via the following expressions

$$P_s = \frac{1}{N} \sum_{k=1}^{N} s_k^2$$

$$P_n = \frac{1}{N} \sum_{k=1}^{N} (\alpha\, n_k)^2\,.$$

Now, we recall our definition of SNR:

$$\text{SNR} = \frac{\sum_{k=1}^{N} s_k^2}{\alpha^2 \sum_{k=1}^{N} n_k^2}$$

Then, we select the value α that yields the desired SNR

$$\alpha^2 = \frac{\sum_{k=1}^{N} s_k^2}{\text{SNR} \sum_{k=1}^{N} n_k^2} = \frac{\|\mathbf{s}\|_2^2}{\text{SNR}\, \|\mathbf{n}\|_2^2}$$

A simple code for adding noise to a signal is provided in the following function:

```
function Add_Noise(s, SNR)

# Compute d = s + n such that SNR = Ps/Pn
#
# s:    Input signal
# SNR:  Desired signal-to-noise ratio
# d:    Output signal

   n = randn(size(s))
   Es = sum(s[:].^2)
   En = sum(n[:].^2)
   alpha = sqrt(Es/(SNR*En))
   d = s+alpha*n
   return d,alpha*n

end
```

Often SNR is given in decibels (dB). In this case

$$\mathrm{SNR}_{\mathrm{dB}} = 10\,\log_{10}(\mathrm{SNR})\,.$$

You can convert $\mathrm{SNR}_{\mathrm{dB}}$ to SNR and then use the code above. If $\mathrm{SNR}_{\mathrm{dB}} = 0$, SNR=1 or in other words, the power of the noise is equal to the power of the clean signal.

Notice that P_n is also an estimator of the variance of the noise σ^2. Therefore, one could have also defined the signal-to-noise ratio as follows

$$\mathrm{SNR} = \frac{P_s}{\sigma^2}\,.$$