Forecasting Recessions in Canada*

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March 13, 2018

Abstract

This paper uses dynamic factors estimated from panels of macroeconomic time series for Canada and the US to forecast probabilities of recession in Canada. The factors are obtained from financial, stock market, and real activity indicators for both countries. We evaluate the predictive content of the estimated factors compared to observed data, as well as U.S. versus domestic data. Our findings show that factor-augmented probit regressions outperform models based solely on observed data, with a real-activity factor performing particularly well at short forecast horizons. In addition, while at longer forecast horizons U.S. interest rate spreads are consistently part of the best performing models, there is little gain in predictive accuracy from adding U.S. macro data. Finally, our findings indicate that BMA forecasts cannot improve upon forecasts from the best individual models.

*We thank participants at the 2014 Canadian Economic Association and the 2015 International Association for Applied Econometrics meetings for useful comments. The views expressed in this paper are solely those of the authors and not necessarily those of the Bank of Canada.

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1 Introduction

Predictions about the state of the economy figure prominently in the decision making process of households, firms, and policy makers. In particular, forecasting recessions (that is, periods of decline in economic activity) is considered to be of special interest in macroeconomics. In the United States (U.S.), recession periods are determined by the Business Cycle Dating Committee of the National Bureau of Economic Research (NBER). Similarly, the Business Cycle Council of the C.D. Howe Institute determines recession periods in Canada. In both countries, however, peak and trough dates are announced with considerable delay. As a result, a common strategy among those interested in modeling the state of the economy in real-time consists in generating probabilities of recession using binary class models (for example, probit or logit models) for current or future recession dates.

In this paper we evaluate the predictability of Canadian recessions using a large data set of Canadian and U.S. macroeconomic indicators and standard probit models. We start estimating dynamic factors from small panels of macroeconomic indicators. Following Ludvigson and Ng (2010) and Fossati (2015), three monthly factors are estimated for each country: (1) a bond and exchange rates factor; (2) a stock market factor; (3) a real activity factor extracted from "coincident" macroeconomic indicators. Next, we construct a dataset of potential predictors that includes the six estimated factors (macro factors) as well as individual indicators that have been found to exhibit good predictive power in the literature and evaluate the predictive performance of different models both in-sample and out-of-sample. Our strategy builds on earlier work by Estrella and Mishkin (1998), Atta-Mensah and Tkacz (1998), Katayama (2010), Hao and Ng (2011), and Fossati (2015), among many others. In particular, we focus our attention on models with and without macro factors and models with and without U.S. data (indicators or factors). By comparing the accuracy of forecasts obtained from models constrained to different subsets of predictors we attempt to answer the following questions: (1) Are forecasts made using estimated factors more accurate than forecasts made using observed indicators directly? (2) Are indicators from the U.S. economy useful to forecasts economic activity in Canada? (3) Are Bayesian model average (BMA) forecasts more accurate than forecasts obtained from individual models?

A rich body of literature examines the predictability of recessions in the U.S. economy, with comparatively little work focusing on Canada.[1] For example, Atta-Mensah and Tkacz (1998) find that the Canadian yield spread, the difference between long term bond yields and the 3-month commercial paper rate, is the most useful indicator to predict recessions in Canada. In addition, Bernard and Gerlach (1998) show that the inclusion of the U.S. yield spread improves recession forecasts at medium and long term horizons. Hao and Ng (2011) expand on the list of indicators used to predict recessions in Canada by considering a small number of financial and real activity indicators (for example, inflation, employment, monthly GDP, and housing starts) as well as dynamic probit models. Finally, Sties (2017) considers a larger data set and uses penalized regression methods to perform variable selection and estimation simultaneously. Recently, the interest has moved to factor-augmented models. For example, Chen *et al.* (2011) and Fossati (2016) employ large factor models to forecast U.S. recessions. In contrast, Fossati (2015) employs small dynamic factors retrieved from panels of financial, stock market and real activity macro variables also to forecast recessions in the U.S. For Canada, to the best of our knowledge, only Gaudreault *et al.* (2003) estimate a dynamic factor model to nowcast recession probabilities.

The main results of this paper can be summarized as follows. First, we find that factor-augmented probit models outperform models estimated with observed data alone. Among the estimated factors, a Canadian real activity factor is particularly successful at predicting turning points at short horizons. On the other hand, we find that financial factors estimated from interest rate spreads and exchange rates improve recession forecasts at 6 to 12 months horizons but only marginally when compared to forecasts made with the observed data alone. Castle *et al.* (2013) find that dynamic factors are better at forecasting GDP at short horizons, while their relative performance declines as the forecast horizon increases. We find that this result holds for recession forecasts in Canada. In terms of the second question, we find that excluding U.S. data results in no substantial deterioration in predictive performance. While at longer forecast horizons U.S. interest rate spreads are consistently part of the best performing models, there is little gain in predictive accuracy from adding U.S. observed predictors and factors. This result contrasts results in Bernard and Gerlach (1998) and Gosselin and Tkacz (2010) who find evidence that U.S. data can be useful to predict recessions and inflation,

[1]See, for example, Berge (2015) and Fossati (2015) and many of the references therein.

respectively. Finally, our findings indicate that BMA forecasts can not improve upon forecasts from the best individual model within the respective subset. This holds true in-sample, where the prediction error of the BMA is similar to the best individual model, but even more so out-of-sample, where the BMA forecasts perform clearly worse than the best individual models.

The remainder of this paper is organized as follows. Section 2 describes the data, the methodology used to estimate the dynamic factors, and the probit regressions used to generate probabilities of recession at different horizons. Section 3 summarizes the estimation results and section 4 concludes the paper.

2 Methodology

In this section we describe the empirical methodology used in this paper. In section 2.1 we discuss the estimation of dynamic factors from six subsets of Canadian and U.S. macroeconomic indicators. Next, we use these estimated factors together with selected individual indicators to generate recession probabilities for the Canadian economy. We present the predictive probit regressions and forecast evaluation statistics in section 2.2. Finally, in section 2.3 we discuss the BMA strategy used to combine forecasts.

2.1 Dynamic Factors

Dynamic latent factors are estimated from 27 different indicators for the Canadian economy and 30 indicators for the U.S. economy. As in Ludvigson and Ng (2010), the data set is organized into six small panels or blocks.[2] For the U.S., we follow Fossati (2015) and consider three panels: (1) a bond and exchange rates data set of 22 financial indicators including interest rates, interest rate spreads, and exchange rates; (2) a data set of 4 stock market indicators including stock price indexes, dividend yield, and price-earnings ratio; (3) a data set of 4 real activity indicators including industrial production, personal income less transfer payments, real manufacturing trade and sales, and employment. Dynamic factors estimated from each of these three panels have been found useful in many forecasting exercises. For example, Ludvigson and Ng (2010) show that an important amount of variation in the two-

[2] See Ludvigson and Ng (2010) for a more detailed motivation to organize the data into blocks.

year excess (U.S.) bond returns can be predicted by factors estimated from panels (1) and (2). Likewise, the real activity variables in panel (3) have been used in Stock and Watson (1991), Diebold and Rudebusch (1996), Kim and Nelson (1998), Chauvet (1998), Chauvet and Piger (2008), Camacho *et al.* (2015), and Fossati (2015, 2016), among others, to model real-time business conditions in the U.S. economy. For Canada we also construct three small panels of Canadian indicators with similar characteristics as those for the U.S. economy. The three panels are: (1) a bond and exchange rates data set of 18 financial indicators including interest rates, interest rate spreads, and exchange rates; (2) a data set of 5 stock market indicators including stock price indexes, dividend yield, and price-earnings ratio; (3) a data set of 4 real activity indicators including housing starts, production in manufacturing, consumer credit, and male employment. In contrast to the U.S. literature described above, the literature on dynamic factors for the Canadian economy is small. For example, Bragoli and Modugno (2017) and Chernis and Sekkel (2017) use real activity dynamic factors estimated using both Canadian and U.S. data to nowcast gross domestic product in Canada. Similarly, Gosselin and Tkacz (2010) use dynamic factors also estimated using both Canadian and U.S. data to forecast the CPI inflation in Canada.

For each of these six panels we estimate a dynamic factor model using Bayesian methods and the following framework. Let x be a $T \times N$ panel of macroeconomic indicators where x_{it}, $i = 1, \ldots, N$ and $t = 1, \ldots, T$, has a factor structure of the form

$$x_{it} = \lambda_i(L)g_t + e_{it} \tag{1}$$

where g_t is an unobserved dynamic factor, $\lambda_i(L) = \lambda_{i0} + \lambda_{i1}L + \cdots + \lambda_{is}L^s$ a polynomial of order s, λ_{ij} are the dynamic factor loadings, and e_{it} the idiosyncratic error. The dynamics of the latent factor and of the idiosyncratic errors are driven by autoregressive processes such that

$$\phi(L)g_t = \eta_t, \qquad \eta_t \sim i.i.d.\, N(0, \sigma_g^2) \tag{2}$$

$$\psi_i(L)e_{it} = \nu_{it}, \qquad \nu_{it} \sim i.i.d.\, N(0, \sigma_i^2) \tag{3}$$

where $\phi(L)$ and $\psi_i(L)$ are polynomials of order p_g and p_e, respectively. The factor model is specified by assuming $s = 2$ and $p_g = p_e = 1$ for every panel so that $\lambda_i(L) = \lambda_{i0} + \lambda_{i1}L + \lambda_{i2}L^2$, $\phi(L) = 1 - \phi L$, and $\psi(L) = 1 - \psi L$ for $i = 1, \ldots, N$. For estimation, the dynamic factor model is written in state-space form and estimated via Gibbs sampling following Kim and Nelson (1999) and Ludvigson and Ng (2010).[3] Identification is achieved by setting $\lambda_{10} = 1$, that is the factor loading on the first time series in each panel to 1. Finally, the parameters λ_{ij} and ψ_i are initialized to zero, σ_g^2, and σ_i^2 are initialized to 0.5, and principal components is used to initialize the dynamic factor. The Gibbs sampler runs 6,000 times. After discarding the first 1,000 draws (burn-in period), posterior means are computed using a thinning factor of 10, that is computed from every 10th draw.

The variables included in each panel, as well as their sources and the transformations employed, are described in the appendix. Our data set starts in 1967:1 and ends in 2016:12. Prior to estimation, the data is transformed to ensure stationarity and standardized. Since real activity variables are usually available with some lag, we account for data availability at time t by using the last known value x_{it-l}, where l indicates the publication lag of variable i. Publication lags for U.S. indicators are adopted from Katayama (2010). Publication lags for Canadian real activity indicators are obtained from Statistics Canada. Figures 1 and 2 depict the three estimated factors from the full sample of Canadian and U.S. data, respectively. The shaded areas indicate recession periods in Canada (Cross and Bergevin, 2012). Both sets of factors display similar characteristics. For example, periods of recession are coincident with dips in the real activity factors and major troughs correspond closely to Canadian recession dates. On the other hand, dips in the financial factors seem to precede recession periods. Finally, the stock market factors are characterized by higher volatility and no obvious correlation with recession months emerges from this plots.

[Figure 1 about here.]

[Figure 2 about here.]

2.2 Predictive Probit Regressions

The recession indicator for the Canadian economy is defined as follows. Let y_{t+h} be a binary variable which equals 1 if the month $t + h$ is subsequently declared as a recession and 0

[3]While the dynamic factors can also be estimated by maximum likelihood, Gibbs sampling provides a more robust alternative for the out-of-sample recursive exercises implemented below.

otherwise. A forecast of the probability of a recession in month $t+h$ (p_{t+h}) from a probit regression is then given by

$$p_{t+h} = P(y_{t+h} = 1 \,|\, z_t) = \Phi(\beta' z_t), \tag{4}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function, β is a vector of coefficients, and z_t is a $k \times 1$ vector of predictors including an intercept. In this paper, we use the business cycle classification provided by Cross and Bergevin (2012) of the C.D. Howe institute.[4]

Our set of potential predictors includes 57 individual indicators and the six dynamic factors estimated from the small panels the individual indicators. To make estimation feasible, we restrict our attention to a subset of individual indicators and the six factors. The selected individual indicators (highlighted with an asterisk in the data appendix) include 14 Canadian indicators (interest rates, exchange rates, interest rate spreads, stock market indexes, and real activity variables) and 5 U.S. indicators (interest rates, interest rate spreads, a stock market index, and industrial production). This set of 18 individual indicators is a mix of variables previously used in the literature, e.g. Hao and Ng (2011), and indicators that are found to be good individual predictors. Finally, we restrict the probit models to a maximum number of three predictors (in addition to an intercept). In total, based on the 24 predictors (including factors), 2324 models are evaluated in this best subset selection exercise.[5]

All models are estimated in-sample as well as recursively out-of-sample. We evaluate the in-sample fit of each model using McFadden's pseudo-R^2 (R^2_{mf}) which is defined as

$$R^2_{mf} = 1 - \frac{\ln \hat{L}_1}{\ln \hat{L}_0}, \tag{5}$$

where $\hat{L}_1$ is the value of the log-likelihood function evaluated at the estimated parameters and $\hat{L}_0$ is the log-likelihood computed only with a constant term. Predicted probabilities of recession, both in-sample and out-of-sample, are evaluated using two popular statistics. The first statistic is the quadratic probability score (QPS), which is equivalent to the mean

[4]We verified the robustness of our results using the recession classification adopted by Atta-Mensah and Tkacz (1998) and Hao and Ng (2011) who extend existing series using a rule of thumb of six months of negative gross domestic product growth. Results are not significantly different to our baseline estimation using the C.D. Howe recession dates.

[5]These 2324 models include 24 one-variable models, 276 two-variable models, and 2024 three-variable models.

squared error and is defined as

$$\text{QPS} = \frac{2}{T^*}\sum_{t=1}^{T^*}(y_{t+h} - \hat{p}_{t+h})^2, \tag{6}$$

where T^* is the effective number of forecasts and $\hat{p}_{t+h} = \Phi(\hat{\beta}' z_t)$ is the predicted probability of recession for month $t+h$ for a given model. The QPS can take values from 0 to 2 and smaller values indicate more accurate predictions. In addition, recession probabilities are also evaluated using the log probability score (LPS) which is defined as

$$\text{LPS} = -\frac{1}{T^*}\sum_{t=1}^{T^*}\left[y_{t+h}\log(\hat{p}_{t+h}) + (1 - y_{t+h})\log(1 - \hat{p}_{t+h})\right]. \tag{7}$$

The LPS can take values from 0 and $+\infty$ and smaller values indicate more accurate predictions. Compared to the QPS, the LPS score penalizes large errors more heavily.

2.3 Bayesian Model Averaging

We use BMA to combine predicted probabilities of recession obtained from the 2324 probit regressions. One of the advantages of BMA is that its forecasts tend to improve accuracy when there is uncertainty about the true model.[6] However, there are few papers exploring BMA in the context of predicting probabilities of recession. For example, Berge (2015) uses model selection and model averaging strategies (including BMA) to evaluate the information content in many economic indicators as predictors of U.S. business cycle turning points. Similarly, Guérin and Leiva-Leon (2014) combine recession probabilities obtained from univariate and multivariate regime-switching models using BMA and other averaging strategies. Both papers find that BMA can yield improvements in forecast accuracy and highlight the importance of allowing for time variation in the models' weights as the best forecasting models typically change over time. In addition, we use the weights assigned to the BMA forecasts to evaluate the predictive content of the dynamic factors vis-a-vis the individual predictors.

The approach we follow to average recession probabilities is similar to Berge (2015). First, from each of the M models estimated in section 2.2 we obtain a forecast $\hat{p}_{t+h}$, resulting in $\{\hat{p}^1_{t+h}, \hat{p}^2_{t+h}, \ldots, \hat{p}^M_{t+h}\}$. The BMA combined forecast assigns each of the M models a weight

[6] See, for example, Faust *et al.* (1996), Wright (2008), and Groen *et al.* (2013), among others.

w_i, $i = 1, \ldots, M$, such that

$$\hat{p}_{t+h}^{BMA} = \sum_{i=1}^{M} \hat{p}_{t+h}^{i} w_i \tag{8}$$

where $w_i = P(M_i \,|\, D)$ is the posterior probability of model i conditional on observed data D. The posterior probability of model i is given by

$$P(M_i \,|\, D) = \frac{P(D \,|\, M_i)P(M_i)}{\sum_{j=1}^{M} P(D \,|\, M_j)P(M_j)} \tag{9}$$

where $P(D \,|\, M_i)$ is the marginal likelihood of model i and $P(M_i)$ is the prior probability that model i is true. Calculating the marginal likelihood can be a high-dimensional and intractable problem. We follow much of the literature and use the BIC approximation as discussed in Raftery (1995). When each model is deemed to be equally likely a priori, the i-th model posterior probability can be approximated by its fit relative to the fit of all other models such that

$$P(M_i \,|\, D) = \frac{\exp(\mathrm{BIC}_i)}{\sum_{i=1}^{M} \exp(\mathrm{BIC}_i)}. \tag{10}$$

As suggested in Raftery (1995), the BIC for model i is defined as

$$\mathrm{BIC}_i = -LR_i + k \ln T \tag{11}$$

where LR_i is the likelihood ratio test statistic for testing model i against a model with only a constant term, k is the number of predictors, and T is the sample size.

3 Results

In this section we compare the predictive performance of the different models, as well as the BMA predictions. In addition, we compare the predictive performance of models that include dynamic factors to models that do not include factors. Similarly, we compare the performance of models that include U.S. data (factors and indicators) to models that include only Canadian data. All models are estimated in-sample (using the full set of available observations), as well as out-of-sample (using only observations up to the time the forecast would have been made to mimic real-time forecasting).

3.1 In-Sample Results

We start by assessing the individual in-sample predictive content of each variable at different forecast horizons. To this end, we estimate one-variable probit models by regressing the recession series y_{t+h} for $h \in \{1, \ldots, 18\}$ on each indicator and estimated factor separately. The models are estimated using data starting in 1967:3 and ending in 2016:12, that is, the full sample. Figure 3 plots the regression R^2_{mf} coefficients versus the forecast horizon h. Gray lines represent the coefficients for the individual indicators while blue lines depict the R^2_{mf} coefficients for the estimated dynamic factors. We present the results following the six panels described above, with models estimated with Canadian data on the left panels and models estimated with U.S. data on the right panels. When comparing the R^2_{mf} coefficients across different forecast horizons we see similar results for U.S. and Canadian indicators and factors, but U.S. variables overall have lower individual predictive content for Canadian recessions. The results show that the predictive content of bond and exchange rate variables is divided into two groups. The four variables with high predictive content peaking around 12-months ahead forecasts are the four Canadian yield spreads. On the other hand, interest rates and exchange rates have low correlation with recessions at any forecast horizon. The financial factor consequently ranges somewhere between the two groups. The stock market indicators have relatively little predictive content. The average predictive content of stock market variables peaks between 3 to 6 months. On the other hand, the real activity indicators have very strong predictive content at short forecast horizons. In particular, notice that the proposed Canadian real activity factor improves significantly upon any of the R^2_{mf} coefficients for the observable indicators.

[Figure 3 about here.]

Next, we estimate all combinations of 3-variable probit models as described in section 2.2. Table 1 reports the in-sample QPS and LPS for the best individual models, as well as the BMA results. Three sets of models can be distinguished. The first set uses the observable indicators and estimated factors, and uses both Canadian and U.S. data ("all variables"). The second set includes models only estimated with the observable indicators, that is, without any of the estimated factors ("w/o factors"). The third set includes all the models estimated using only

Canadian data, that is, without any U.S. data ("w/o U.S. variables"). All variables are for the Canadian economy unless indicated otherwise. For $h = 1$, the shortest forecast horizon considered, the best model (908) according to the QPS criterion, includes housing starts (HS), the 5-year Canadian yield spread ($YS5$), and the Canadian real activity factor ($real^{CA}$). If factors are excluded from the set of predictors, the best model (1950) uses consumer credit (CC), male employment (EMP), and the 5-year yield spread (YS_5). Excluding the Canadian real activity factor, however, results in a substantial deterioration in fit (larger QPS and LPS values). Finally, since the best model includes only Canadian data, for $h = 1$ the selected model does not change when U.S. variables are excluded.

When the forecasting horizon is increased to 3 months ($h = 3$) we find that the best performing models are very similar to the models for $h = 1$, with the only difference that 10-year Canadian government bond yields replace (GB10) replace the housing starts variable. The models, however, exhibit a small deterioration in fit due to the reduced predictive content of the Canadian real activity factor at longer horizons. In contrast, as the forecasting horizon is increased to 6 and 12 months, the best performing models change and U.S. variables start appearing. For $h = 6$, the best performing model (891) now includes the 10-year Canadian yield spread ($YS10$), the USD-Canadian dollar exchange rate ($EXUS$), and the Canadian real activity factor ($real^{CA}$). For $h = 12$, the best performing model (1034) drops the Canadian real activity factor and incorporates the U.S. financial factor (fin^{US}) as well as the Bank of Canada's Bank rate. As a result, for 6 and 12 month ahead forecasts, the performance of the best models deteriorates slightly when U.S. variables are excluded from the potential set of predictors, with the U.S. financial factor (fin^{US}) being replaced by the Canadian financial factor (fin^{CA}). When the dynamic factors are excluded we observe a large deterioration in the forecasting performance at 1, 3, and 6 months, but a relatively small improvement at the 12 months horizon.

[Table 1 about here.]

Next, we focus on the in-sample performance of the BMA forecasts. The results reported in Table 1 show that BMA delivers an in-sample performance that is essentially identical to the one reported for the best performing models. For $h = 1$, Figure 4 shows that about 50% weight is given to the best performing model (908), with the other 50% being allocated to slightly different model in close vicinity to model (908) indicating that only the model's third variable, housing starts, changes. For $h = 3$, the same model receives about 33% weight, with another 30% weight being allocated to model 860, the best performing individual model for this horizon. Similarly, for $h = 6$, the model with the highest weight (133) of 60% is a slight variation of the best performing model (891) with the Canadian real factor substitutes by the U.S. real factor. Finally, for $h = 12$ we find that 55% of the weight is given to model (617), again, a variation of the best performing model (1034). Several conclusions can be drawn from the in-sample BMA results. First, at all horizons, the model being allocated the most weight is always similar to the best performing model according to QPS. Second, BMA gives positive weight to few models, and these models are generally very similar models. As a result, BMA weights are highly concentrated on few very effective predictors and BMA forecasts end up being very similar to the ones obtained from the best forecasting models.

[Figure 4 about here.]

In sum, our in-sample results show that Canadian real activity indicators (housing starts and employment) and particularly the Canadian real activity factor are the preferred variables for generating short term (1 to 3 months) recession probabilities of the Canadian economy. At longer horizons (6 to 12 months), the preferred variables include Canadian yield spreads, mainly the 10-year yield spread. In terms of the questions formulated in the introduction, we find the following results: (1) Excluding U.S. data results in a very small deterioration in fit at longer horizons; (2) Excluding factors can result in a substantial deterioration in fit at shorter horizons; (3) BMA forecasts cannot improve the performance of the best model selected by QPS. Finally, to illustrate point (2), Figure 5 shows the in-sample predicted probabilities of recession for the best performing models with and without factors. For forecast horizons of 1 and 3 months, the best model with factors produces recession probabilities that are closer to 0 during expansions and closer to 1 during recessions. This improvement, however, vanishes as the forecasts horizon is extended to 6 and 12 months.

[Figure 5 about here.]

3.2 Out-of-Sample Results

We now evaluate the performance of the models in a recursive out-of-sample forecasting exercise.[7] In this case, the set of observations is divided into an initial estimation sample from 1961:4 to 1979:12 ($225 - h$ effective observations) and a hold-out sample with the remaining 444 observations. A direct h-step ahead forecast is produced for each period in the hold-out sample, with the first forecast made for 1980:1+h and the last for 2016:12. As a result, the hold-out sample includes 443 out-of-sample predictions when $h = 1$, 441 predictions when $h = 3$, 438 predictions when $h = 6$, and 432 predictions when $h = 12$. First, the dynamic factors are estimated recursively, each period using data available at time t, and expanding the estimation window by one observation each month. Next, the probit models are also estimated recursively and used to generate a recession probability for month $t + h$ based the information available at month t. We account for data availability at each point in time by adjusting for the publication lag in real activity variables (see, for example, Katayama, 2010; Fossati, 2015).

Table 2 reports the out-of-sample QPS and LPS for the best individual models, as well as the BMA results. For $h = 1$, the best model (827) includes the real activity factors for Canada and the U.S. ($real^{CA}$,$real^{US}$) as well as the 10-year Canadian yield spread (YS_{10}). At longer horizons, the observations made in-sample also largely translate to the out-of-sample results but with some differences. For example, U.S. variables now appear more often and at shorter forecast horizons. U.S. industrial production ($INDPRO^{US}$) is selected at $h = 1$ when factors are excluded, the U.S. real activity factor ($real^{US}$) is selected at $h = 1$ and $h = 6$, the U.S. stock market factor ($stock^{US}$) is selected at $h = 12$, when factors are excluded, it is replaced by the S&P 500 index ($SP500^{US}$). But while U.S. variables appear to be more relevant, excluding U.S. variables from the potential set of predictors has almost no effect in the out-of-sample performance of the models (mainly larger LPS values at $h = 6$). On the other hand, at shorter forecast horizons we find that factors improve the out-of-sample performance of the models and are selected more often. Consequently, excluding the estimated factors from the set of predictors results in a larger deterioration in fit than what we observed in the in-sample results.

[7]This exercise uses ex-post revised data (instead of real-time data) to generate out-of-sample predicted recession probabilities for each of the models.

[Table 2 about here.]

We now focus on the out-of-sample performance of the BMA forecasts. One advantage of averaging is that BMA weights are re-computed for each period in the hold-out sample. As a result, the models and therefore the variables that are good predictors are allowed to change over time. Figure 6 shows each of the model's weight for each of the out-of-sample predictions. For example, for the 1-month forecast, between the recessions in the early 1980's and the next recession in the early 1990's, the dominant model in the BMA forecast (blue line) is model 907, including the Canadian real activity factor ($real^{CA}$), the U.S. real activity factor ($real^{US}$), and the Canadian 10-year yield spread ($YS10$). After a period model weight have high variability, after the 2008/2009 recession only two models remain dominant. The yellow line indicates model 908 ($real^{CA}, YS5, HS$), which was the best performing model in sample according to QPS. Only within the last few observations, model 895 ($real^{CA}, YS10, PRODMAN$) receives a slightly higher weight, however, in the last observation, the highest weight is again attributed to model 908, which naturally is the model that received the highest BMA weight in the in-sample (full sample) estimation.

The out-of-sample BMA weights for other forecast horizons paint a similar picture. For the 3-month forecasts, BMA allocates most weight to the same variables as 1-month ahead. For the 6-month forecasts, the models with highest weight include the Canadian real activity factor, the 10-year yield spread, and housing starts as the most selected predictors. Finally, the 12-month ahead forecasts include variables such as the Canadian and U.S. financial factors, as well as Canadian and U.S. yield spreads. At each horizon, all other models receive very low weights throughout the entire hold-out sample period. As a result, although the dominant model changes over time, essentially the same set of variables is selected consistently for each forecast horizon.

[Figure 6 about here.]

In terms of the recursive out-of-sample performance of the BMA forecasts, the results reported in Table 2 show that averaging cannot improve the accuracy of the best models selected by QPS. In fact, averaging can result in a substantial deterioration in accuracy at longer horizons. Overall, the out-of-sample results are consistent with the in-sample results discussed above and show that real activity variables (the Canadian and U.S. real activity factors, housing starts, etc.) are the preferred variables for generating short term (1 to 3 months) recession probabilities of the Canadian economy. At longer horizons (6 to 12 months), the preferred variables include the Canadian and U.S. financial factors, as well as yield spreads. In terms of the questions formulated in this paper, we find the following results: (1) Excluding U.S. data results in no substantial deterioration in out-of-sample fit; (2) Excluding factors results in large deterioration in fit at shorter horizons and smaller deteriorations at longer horizons; (3) BMA forecasts cannot improve the performance of the best model selected by QPS. Finally, Figure 7 shows the out-of-sample predicted probabilities of recession for the best performing models with and without factors. For forecast horizons of 1 and 3 months, the best model with factors produces produces recession probabilities that are closer to 0 during expansions and closer to 1 during recessions. On the other hand, almost no improvements are observed for forecast horizons of 6 and 12 months.

[Figure 7 about here.]

4 Conclusion

In this paper we evaluated the predictability of recessions in Canada using a large panel of macroeconomic time series for Canada and the U.S. Our findings confirm the importance of domestic yield spreads in making predictions at any forecast horizon as yield spreads appear in some form in every model selected by best subset selection. Yield spreads are best complemented with real activity indicators at short forecast horizons and with financial indicators at long forecast horizons. Stock market indicators generally do not exhibit strong predictive content at any forecast horizon.

We also evaluated the predictive content of six estimated dynamic factors compared to observed data. The factors were obtained from financial, stock market, and real activity indicators for both countries. Our findings show that factor-augmented probit regressions outperform models based solely on observed data. In particular, a real-activity factor performs particularly well at short forecast horizons. Our factor can be used as a coincident indicator due to its strong correlation with the Canadian business cycle. U.S. data appears to improve recession forecasts at longer horizons. This is in line with the notion that spillovers from the U.S. affect economic conditions in Canada with a delay (Beaton *et al.*, 2014). Our findings are similar to the results of Bragoli and Modugno (2017) who find that U.S. variables matter when nowcasting Canadian GDP and confirm the finding of Bernard and Gerlach (1998) that U.S. indicators add predictive power to Canadian recession forecasts at medium and long forecast horizons, but not at shorter ones. Finally, our findings indicate that BMA forecasts can not improve upon forecasts from the best individual models. In particular, out-of-sample BMA forecasts perform clearly worse than the best individual models.

Data Appendix

The following tables list the short name, transformation applied, and a data description of each series in the six groups considered. Canadian data are retrieved from the statistics Canada CANSIM database as well as the OECD. All U.S. bond, exchange rates, and stock market series are from FRED (St. Louis Fed), unless the source is listed as GFD (Global Financial Data), or AC (author's calculation). Data for the U.S. real activity factor are from Camacho *et al.* (2015). The transformation codes are: 1 = no transformation; 2 = first difference; 3 = first difference of logarithms.

[Table 3 about here.]

[Table 4 about here.]

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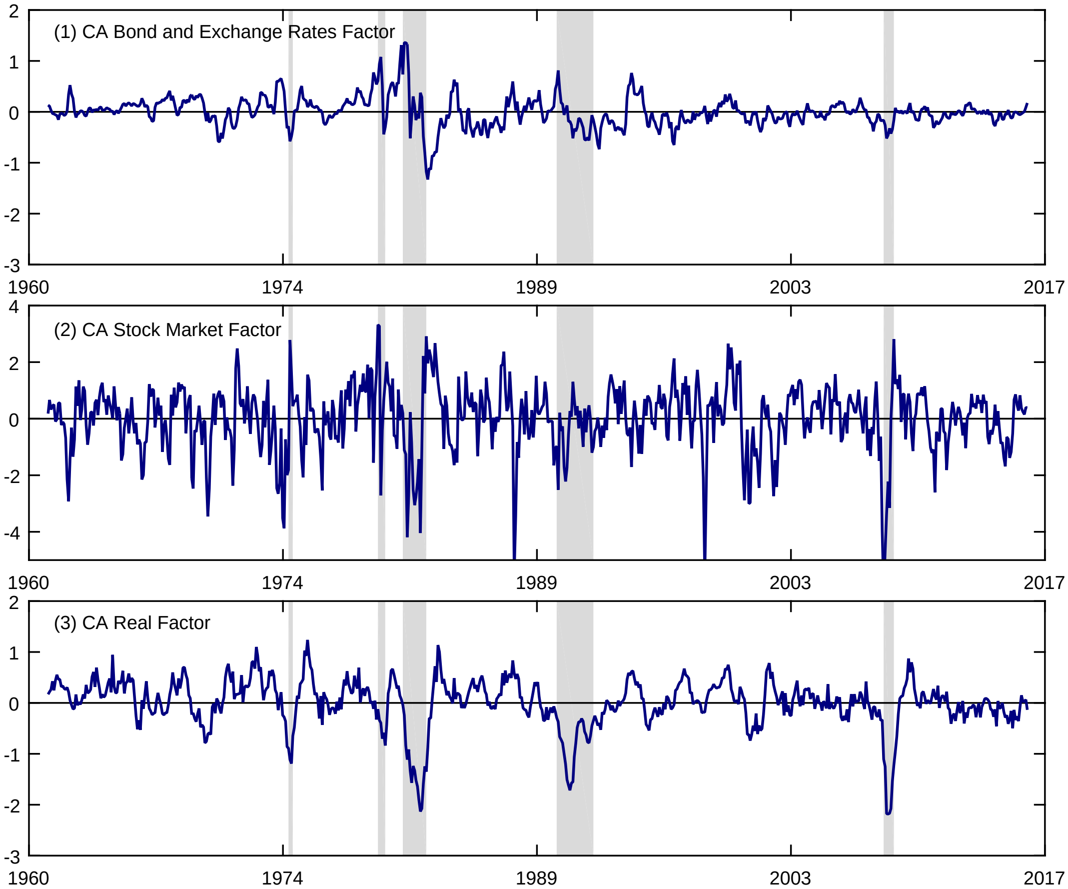

Figure 1: Full sample estimates (posterior means) of the CA dynamic factors. Shaded areas denote recession months in Canada according to the chronology of the C.D. Howe Institute (Cross and Bergevin, 2012).

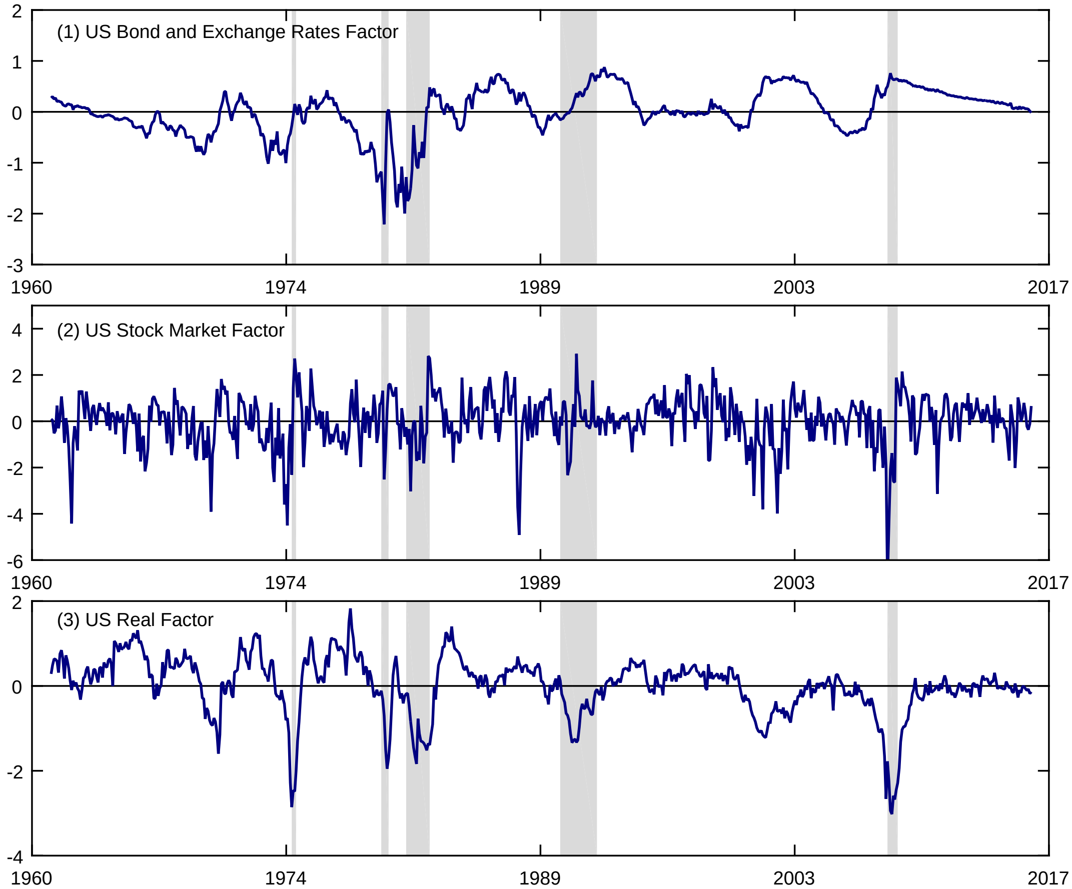

Figure 2: Full sample estimates (posterior means) of the U.S. dynamic factors. Shaded areas denote recession months in Canada according to the chronology of the C.D. Howe Institute (Cross and Bergevin, 2012).

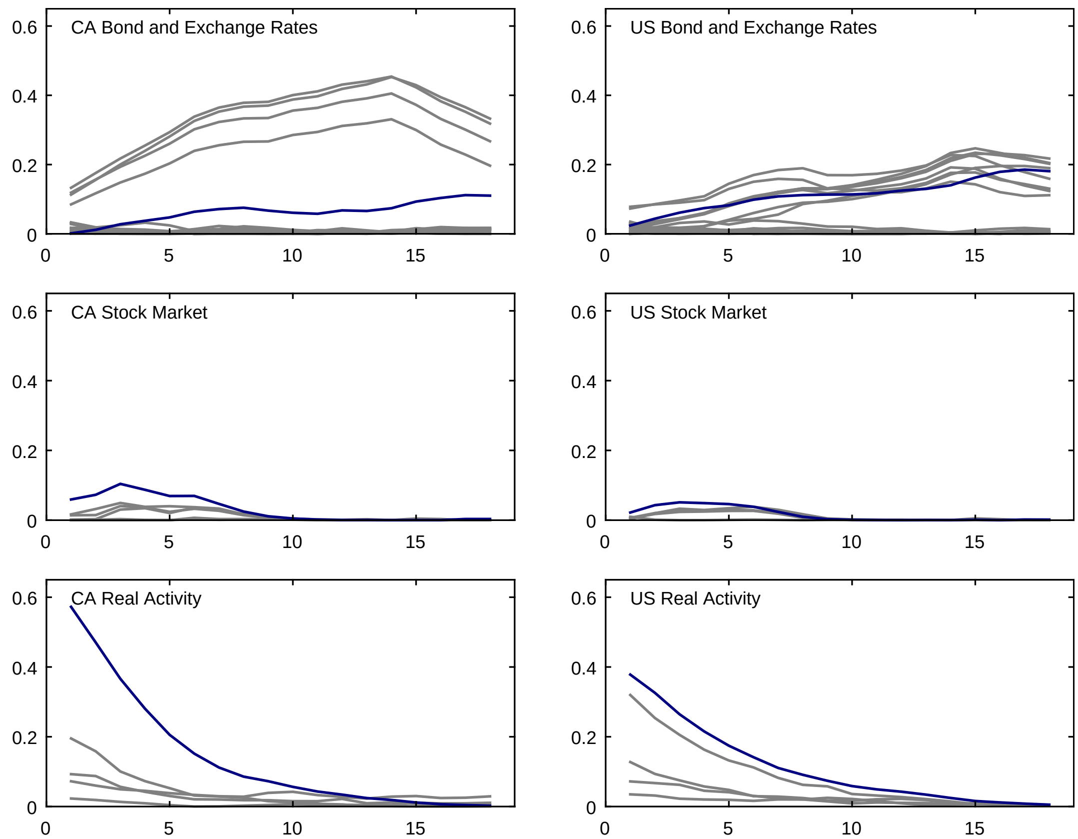

Figure 3: In-sample predictive content of Canadian (CA) and U.S. indicators and factors. Gray lines indicate R^2_{mf} coefficients of observable predictors, blue lines indicate R^2_{mf} coefficients of the dynamic factor estimated from the corresponding group of indicators.

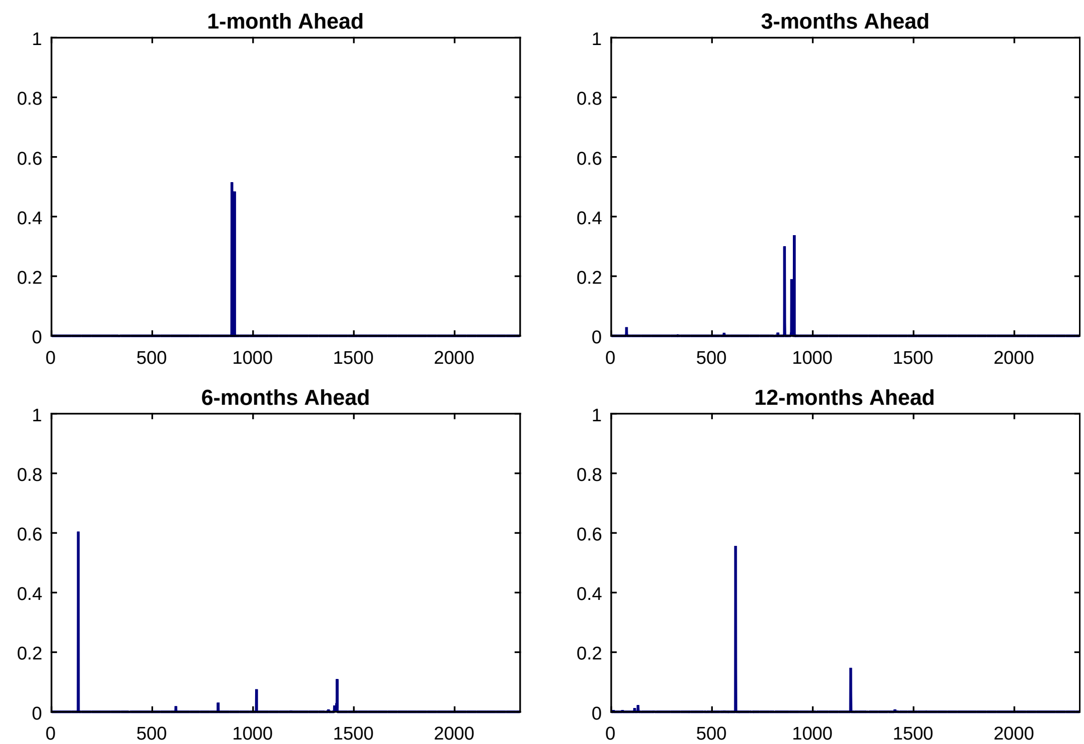

Figure 4: In-sample BMA weights for each of the 2625 3-variable probit models at different forecast horizons.

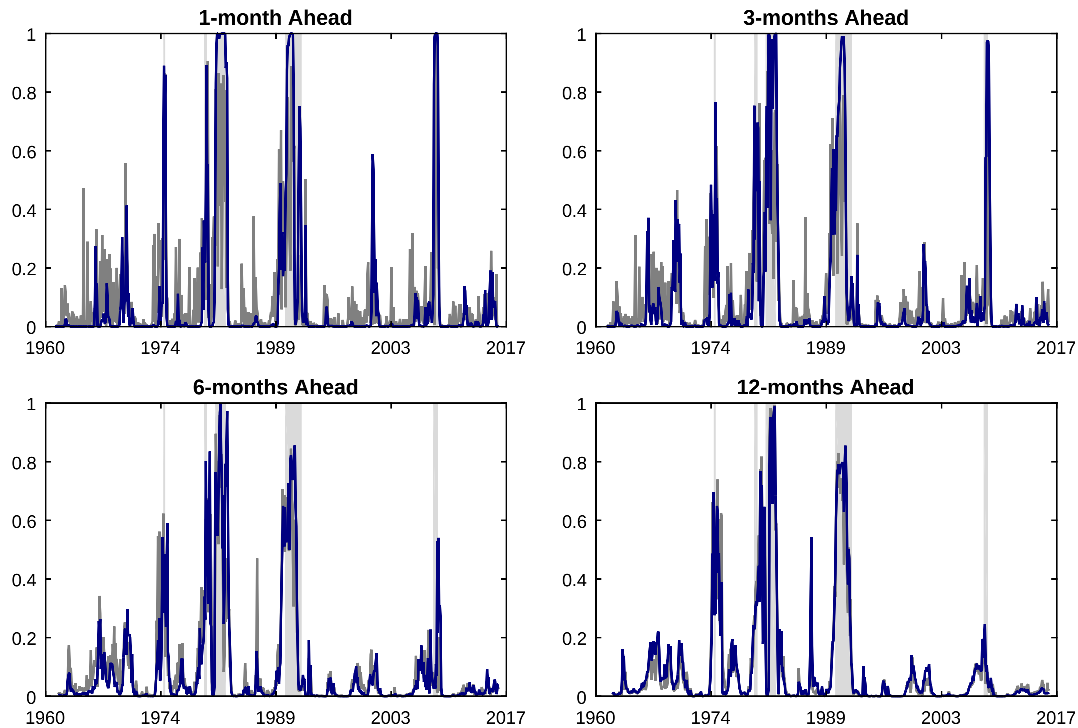

Figure 5: In-sample predicted probabilities of recession for the best performing 3-variable probit models at different forecast horizons: with factors (blue); without factors (gray).

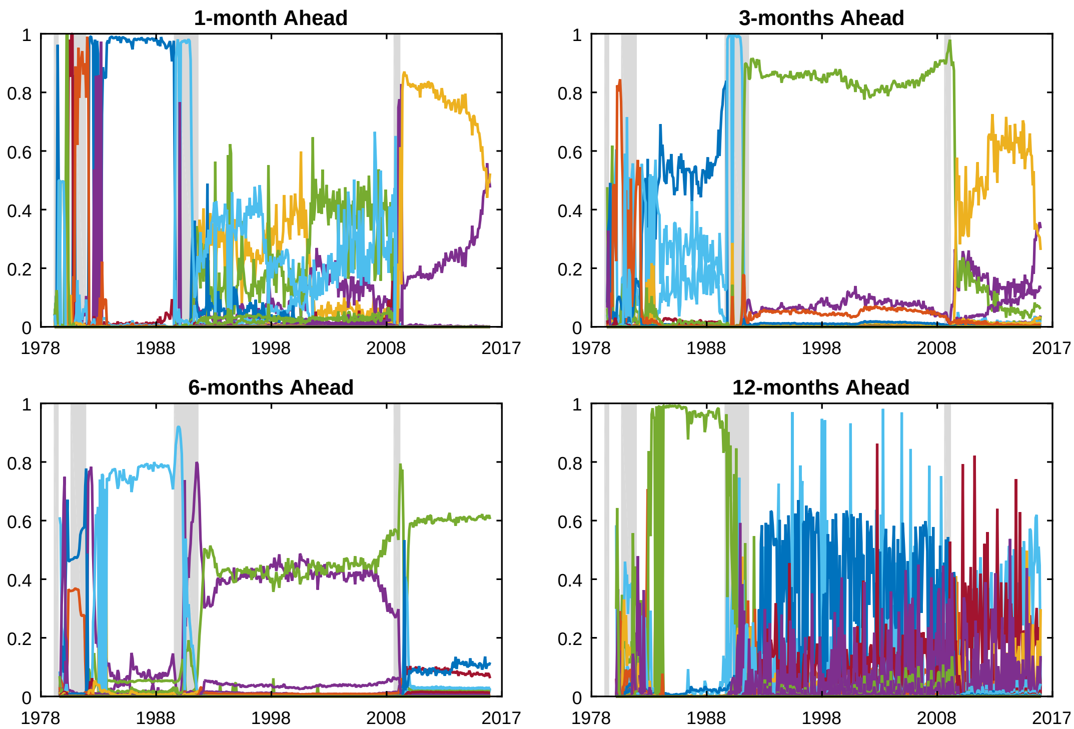

Figure 6: Out-of-sample BMA weights for each of the 2625 3-variable probit models estimated recursively at different forecast horizons.

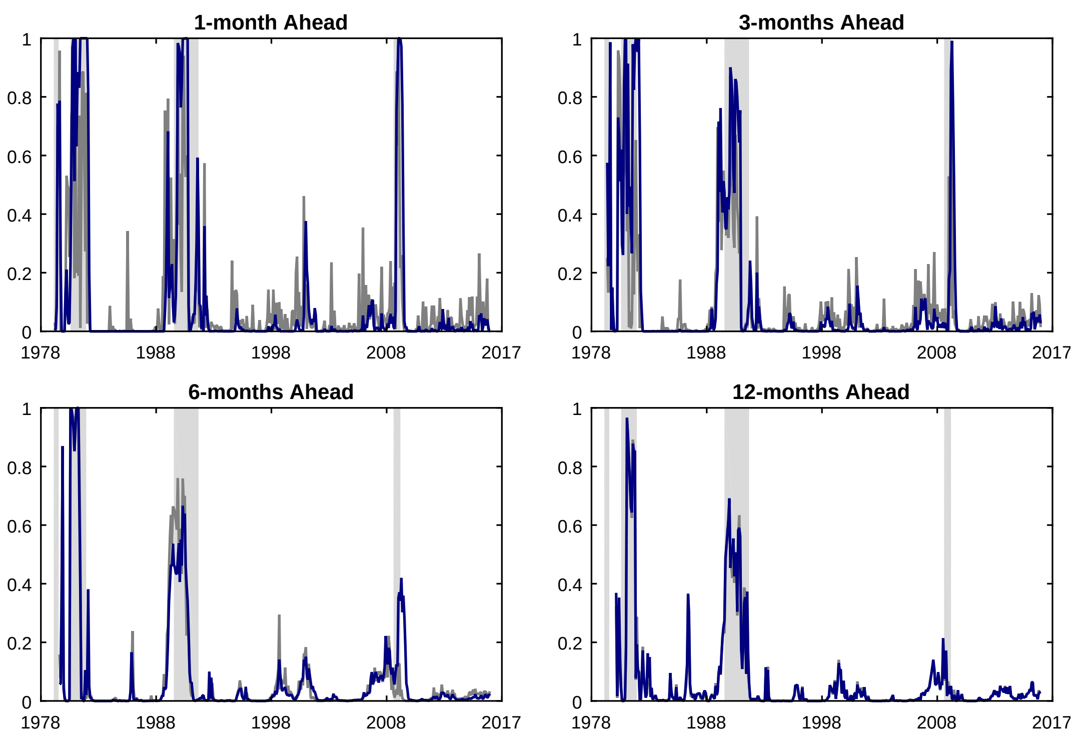

Figure 7: Out-of-sample predicted probabilities of recession for the best performing 3-variable probit models at different forecast horizons: with factors (blue); without factors (gray).

Table 1: Model comparison of in-sample results

$h=1$	all variables Best	all variables BMA	w/o factors Best	w/o factors BMA	w/o U.S. variables Best	w/o U.S. variables BMA
Model	908	–	1950	–	908	–
Var1	$real^{CA}$	–	$YS10$	–	$real^{CA}$	–
Var2	$YS5$	–	$EMPM$	–	$YS5$	–
Var3	HS	–	$INDPRO^{US}$	–	HS	–
QPS	0.05	0.05	0.101	0.101	0.05	0.05
LPS	0.089	0.089	0.186	0.185	0.089	0.089
T	668	668	668	668	668	668
	all variables		w/o factors		w/o U.S. variables	
$h=3$	Best	BMA	Best	BMA	Best	BMA
Model	860	–	1950	–	860	–
Var1	$real^{CA}$	–	$YS10$	–	$real^{CA}$	–
Var2	$GB10$	–	$EMPM$	–	$GB10$	–
Var3	$YS10$	–	$INDPRO^{US}$	–	$YS10$	–
QPS	0.073	0.072	0.11	0.111	0.073	0.072
LPS	0.128	0.125	0.197	0.193	0.128	0.125
T	666	666	666	666	666	666
	all variables		w/o factors		w/o US variables	
$h=6$	Best	BMA	Best	BMA	Best	BMA
Model	891	–	1548	–	891	–
Var1	$real^{CA}$	–	BR	–	$real^{CA}$	–
Var2	$YS10$	–	$YS10$	–	$YS10$	–
Var3	$EXUS$	–	$EMPM$	–	$EXUS$	–
QPS	0.09	0.091	0.104	0.103	0.09	0.093
LPS	0.164	0.156	0.192	0.184	0.164	0.165
T	663	663	663	663	663	663
	all variables		w/o factors		w/o U.S. variables	
$h=12$	Best	BMA	Best	BMA	Best	BMA
Model	1034	–	1565	–	419	–
Var1	fin^{US}	–	BR	–	fin^{CA}	–
Var2	BR	–	$YS5$	–	$GB10$	–
Var3	$YS10$	–	$SP500^{US}$	–	$YS10$	–
QPS	0.093	0.097	0.097	0.099	0.094	0.098
LPS	0.173	0.161	0.17	0.171	0.168	0.168
T	657	657	657	657	657	657

Notes: Selected variables and goodness of fit measures of best individual model selected by QPS and BMA forecast obtained from in-sample forecasting. Lower case variables refer to estimated factors, upper case variables to individual indicators. U.S. variables and factors are denoted with superscript.

Table 2: Model comparison of out-of-sample results

$h=1$	all variables		w/o factors		w/o U.S. variables	
	Best	BMA	Best	BMA	Best	BMA
Model	827	–	1950	–	329	–
Var1	$real^{CA}$	–	$YS10$	–	fin^{CA}	–
Var2	$real^{US}$	–	$EMPM$	–	$real^{CA}$	–
Var3	$YS10$	–	$INDPRO^{US}$	–	$YS10$	–
QPS	0.108	0.124	0.145	0.153	0.11	0.125
LPS	0.217	0.364	0.28	0.334	0.23	0.333
T	443	443	443	443	443	443

$h=3$	all variables		w/o factors		w/o U.S. variables	
	Best	BMA	Best	BMA	Best	BMA
Model	875	–	1934	–	875	–
Var1	$real^{CA}$	–	$YS10$	–	$real^{CA}$	–
Var2	$M1$	–	$PROMAN$	–	$M1$	–
Var3	$YS10$	–	$EMPM$	–	$YS10$	–
QPS	0.14	0.166	0.164	0.18	0.14	0.16
LPS	0.294	0.404	0.309	0.354	0.294	0.389
T	441	441	441	441	441	441

$h=6$	all variables		w/o factors		w/o US variables	
	Best	BMA	Best	BMA	Best	BMA
Model	617	–	1543	–	634	–
Var1	$stock^{CA}$	–	BR	–	$stock^{CA}$	–
Var2	$real^{US}$	–	$YS10$	–	BR	–
Var3	$YS10$	–	$TCIC$	–	$YS10$	–
QPS	0.132	0.151	0.138	0.145	0.132	0.152
LPS	0.297	0.355	0.334	0.348	0.335	0.358
T	438	438	438	438	438	438

$h=12$	all variables		w/o factors		w/o U.S. variables	
	Best	BMA	Best	BMA	Best	BMA
Model	1270	–	2016	–	1965	–
Var1	$stock^{US}$	–	$YS5$	–	$YS5$	–
Var2	$YS5$	–	$PROMAN$	–	$EXUS$	–
Var3	$PROMAN$	–	$SP500^{US}$	–	$PROMAN$	–
QPS	0.119	0.162	0.12	0.135	0.122	0.157
LPS	0.239	0.337	0.247	0.296	0.242	0.32
T	432	432	432	432	432	432

Notes: Selected variables and goodness of fit measures of best individual model selected by QPS and BMA forecast obtained from recursive out-of-sample forecasting. Lower case variables refer to estimated factors, upper case variables to individual indicators. U.S. variables and factors are denoted with superscript.

CA Variables

	Short Name	Trans.	Description
Bond and Exchange Rates Factor			
1*	BR	2	Bank rate (Percent)
2*	GB10	2	Governement marketable bonds average yield (over 10 years)
3	GB5	2	Governement marketable bonds average yield(5-10 years)
4	GB3	2	Governement marketable bonds average yield (3-5 years)
5	GB1	2	Governement marketable bonds average yield (1-3 years)
6	PCP3	2	3 months prime corporate paper
7	PCP2	2	2 month prime corporate paper
8	PCP1	2	1 month prime corporate paper
9	MLR	2	Average residential mortgage lending rate: 5 year
10*	M1	3	Narrow Money (M1) Index 2005=100; SA
11*	YS10	1	Yield Spread b/t 10-yr bond and 3-m prime (AC)
12*	YS5	1	Yield Spread b/t 5-10-yr bond and 3-m prime (AC)
13	YS3	1	Yield Spread b/t 3-5-yr bond and 3-m prime (AC)
14	YS1	1	Yield Spread b/t 1-3-yr bond and 3-m prime (AC)
15*	EXUS	2	Exchange Rate United States dollar, noon spot rate, average
16	EXJAP	2	Exchange Rate Japanese yen, noon spot rate, average
17	EXSWIT	2	Exchange Rate Swiss franc, noon spot rate, average
18*	EXUK	2	Exchange Rate United Kingdom pound sterling, noon spot rate, average
Stock Market Factor			
29*	TCIC	3	TSX Composite Index; Close (2000=1000)
20	STYC	2	Exchange;stockyields(composite);closingquotations(Percent)
21*	SP	3	Share Prices; Index 2005=100
22	TSXVAL	3	Toronto Stock Exchange, value of shares traded (x 1,000,000)
23	TSXVOL	3	Toronto Stock Exchange, volume of shares traded (shares x 1,000,000)
Real Factor			
24*	HS	1	Housing starts index; total units
25*	PRODMAN	3	Production in total manufacturing sa; 2005=100
26*	CC	3	Consumer Credit; At month-end; sa ; Total outstanding balances
27*	EMPM	2	Employed population; Aged 15 and over; Males

U.S. Variables

	Short Name	Trans.	Description
Bond and Exchange Rates Factor			
1*	FEDFUNDS	2	Interest Rate: Federal Funds (Effective) (% per annum)
2	CP3Mx	2	Commercial Paper Rate
3	TB3MS	2	USTreasury Bills, Sec Mkt, 3-Mo. (% per annum)
4	TB6MS	2	USTreasury Bills, Sec Mkt, 6-Mo. (% per annum)
5	GS1	2	USTreasury Const Maturities, 1-Yr. (% per annum)
6	GS5	2	USTreasury Const Maturities, 5-Yr. (% per annum)
7	GS10	2	USTreasury Const Maturities, 10-Yr. (% per annum)
8	AAA	2	Bond Yield: Moody's AAA Corporate (% per annum) (GFD)
9	BAA	2	Bond Yield: Moody's BAA Corporate (% per annum) (GFD)
10	COMPAPFFx	1	Yield Spread b/t Comm paper and Fed Funds (AC)
11	TB3SMFFM	1	Yield Spread b/t 3-m T-bill and Fed Funds (AC)
12	TB6SMFFM	1	Yield Spread b/t 6-m T-bill and Fed Funds (AC)
13	T1YFFM	1	Yield Spread b/t 1-y T-bond and Fed Funds (AC)
14*	T5YFFM	1	Yield Spread b/t 5-y T-bond and Fed Funds (AC)
15*	T10YFFM	1	Yield Spread b/t 10-y T-bond and Fed Funds (AC)
16	AAAFFM	1	Yield Spread b/t AAA bond and Fed Funds (AC)
17	BAAFFM	1	Yield Spread b/t BAA bond and Fed Funds (AC)
18	TWEXMMTH	3	Exchange Rate Index (Index No.) (GFD)
19	EXSZUSx	3	Exchange Rate Switzerland (Swiss Franc per US$)
20	EXJPUSx	3	Exchange Rate Japan (Yen per US$)
21	EXUSUKx	3	Exchange Rate United Kingdom (Cents per Pound)
22	EXCAUSx	3	Exchange Rate Canada (Canadian$ per US$)
Stock Market Factor			
23*	S&P 500	3	S&P's Common Stock Price Index: Composite (1941-43=10) (GFD)
24	S&P indst	3	S&P's Common Stock Price Index: Industrials (1941-43=10) (GFD)
25	S&P div yield	3	S&P's Composite Common Stock: Dividend Yield (% per annum) (GFD)
26	S&P PE ratio	3	S&P's Composite Common Stock: Price-Earnings Ratio (%) (GFD)
Real Factor			
27*	INDPRO	3	Industrial Production Index - Total Index
28	W875RX1	3	Personal Income Less Transfer Payments
29	CMRMTSPLx	3	Manufacturing and Trade Sales
30	PAYEMS	3	Employees On Nonfarm Payrolls: Total Private