A NEW JOINT P AND S VELOCITY MODEL OF THE MANTLE PARAMETERIZED IN CUBIC B-SPLINES

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Abstract

The objective of the project is to develop and improve upon three-dimensional (3-D) seismic velocity models of the Earth and to utilize such models for improving the locations of events recorded at regional and teleseismic distances. Our previous experiments performed using 3-D models of varying resolution show that the accuracy of event location with respect to ground truth does not necessarily improve with an increasing number of free parameters in a model. One reason for this may be the lack of waveform data used for constructing recent high-resolution models, since the maximum sensitivity of seismic travel times to structure is in the middle to lower mantle. In this paper we develop a new joint, P and S velocity model parameterized in terms of radial and horizontal cubic splines, using a combination of direct and differential travel times and surface wave phase measurements. The accuracy of the travel time data has been improved by relocating the events using a previously existing 3-D mantle P wave model. The cubic spline parameterization conveniently allows more detailed models to be inserted within it for use of regional phases in event location. Our preliminary model shows substantial differences in compressional velocity in the uppermost mantle compared with previous models. In conjunction with the model development, we are collecting new reference events with accurate locations, among them earthquakes on mid-ocean ridges and transforms, using a constrained inversion technique.

Key words: Event location; seismic tomography; mantle heterogeneity

Objective

The aim of this project is to ultimately improve the locations of earthquakes and other seismically recorded events in order to enhance the ability to monitor the Comprehensive Nuclear-Test-Ban-Treaty (CTBT). Our strategy is based on developing new, detailed 3-D models of the mantle, with an emphasis on P wave structure. This involves the construction of global high-resolution models with more detail in certain areas where particularly good data coverage is available. A second, subsidiary objective concerns the development of additional techniques used in locating events teleseismically and regionally with sparse datasets, and with assessing the improvement in accuracy afforded by the new techniques and models. We have previously studied the location accuracy obtained by some previously existing mantle P wave models [*Antolik et al.*, 2000]. Here we describe the development of a new joint P and S wave velocity model of the mantle, using a diverse and improved data set of travel times and surface wave phase velocity measurements.

Research Accomplished

The accuracy of event location with respect to ground truth is not a simple function of the number of free parameters or "resolution" of a particular velocity model. This was reported by *Antolik et al.* [2000], where they tested four existing P wave models. Two of these models, S&P12/WM13 (hereafter referred to as SP12) [*Su and Dziewonski*, 1993] and MK12WM13 [*Su and Dziewonski*, 1997] were parameterized in terms of relatively low-order spherical harmonic functions. The other two models, that of *van der Hilst et*

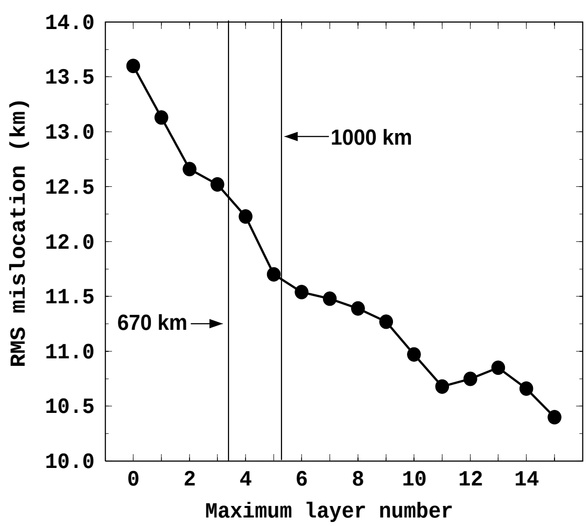

Figure 1: Root mean square mislocation obtained from locating a set of nuclear explosions with known locations using model corrections from BDP98 corresponding to layers only shallower than a certain depth. Each layer is slightly less than 200 km thick and are numbered in ascending order from the top to the bottom of the mantle. The depths of 670 km and 1000 km are indicated by the vertical lines. The data point for layer 0 is equivalent to locating the events in the reference 1-D model PREM (no layers from BDP98 were used). The majority of the improvement in location accuracy is obtained by considering only those layers above 1000 km.

al. [1997] (HWE97) and *Boschi and Dziewonski* [1999] (BDP98), were parameterized in terms of constant-velocity blocks with horizontal dimensions of 2° and 5°, respectively, and contain nearly an order of magnitude more free parameters. Surprisingly, the best locations were provided by model SP12 even though the models with higher resolution provide a better fit to the travel time residuals. These tests were performed using a well-calibrated set of explosions and earthquakes [*Kennett and Engdahl*, 1991] and datasets consisting of a limited number of teleseismic phases in order to simulate global location of small events by the International Monitoring System (IMS) seismic network. Model SP12 provided consistently smaller magnitude mislocations than the other three models.

There are several possible reasons for this result. The parameterization most often chosen for newer, high-resolution models is typically that of blocks with constant velocities [e.g., *Boschi and Dziewonski*, 1999; *Vasco and Johnson*, 1998; *Grand et al.*, 1997]. These models therefore introduce artificial lateral discontinuities into the structure and may induce an unrealistic shape in long-wavelength anomalies. A lack of correlation between new, high-resolution Earth models and earlier longer wavelength models has previously been noted [*Grand et al.*, 1997]. Another issue is the coverage of the data sets used in the inversion problem. Because of the very large number of parameters used to define the higher resolution models, most of these models make use of a somewhat limited data set (for example, only phase travel times) in order to reduce the size of the necessary computer resources (memory and CPU time). This may result in a relative lack of resolution in certain areas of the mantle (particularly at shallow depths) to which teleseismic phase travel times are not sensitive. As an example, we show in Figure 1 the mislocation obtained from a group of explosions using only a subset of the layers of model BDP98 as a correction to PREM. Nearly all of the improvement in the locations is obtained by considering depths above 1000 km, which shows that resolution of the shallow mantle is most critical to teleseismic event location. Models which do not recover the upper mantle structure well will not provide the most accurate locations.

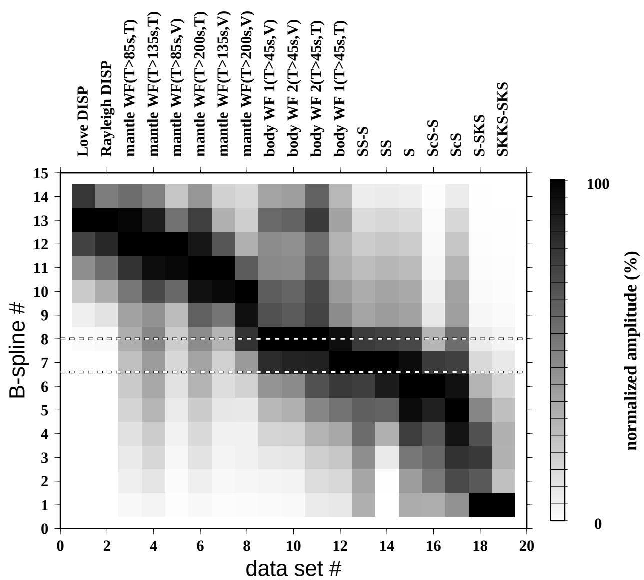

Figure 2: Diagram showing the average amplitide of the diagonal elements of an inner product matrix for an inversion for shear wave velocity [*Gu et al.*, 2000]. Each cell corresponds to that portion of the matrix describing sensitivity of a particular data set used (listed at the top) to one of the radial B-splines of the model. The splines are numbered in ascending order from the CMB to the top of the mantle (i.e., spline 1 has maximum amplitude near the CMB, see Figure 4). A higher normalized amplitude indicates greater sensitivity of a dataset to the coefficients of a radial spline.

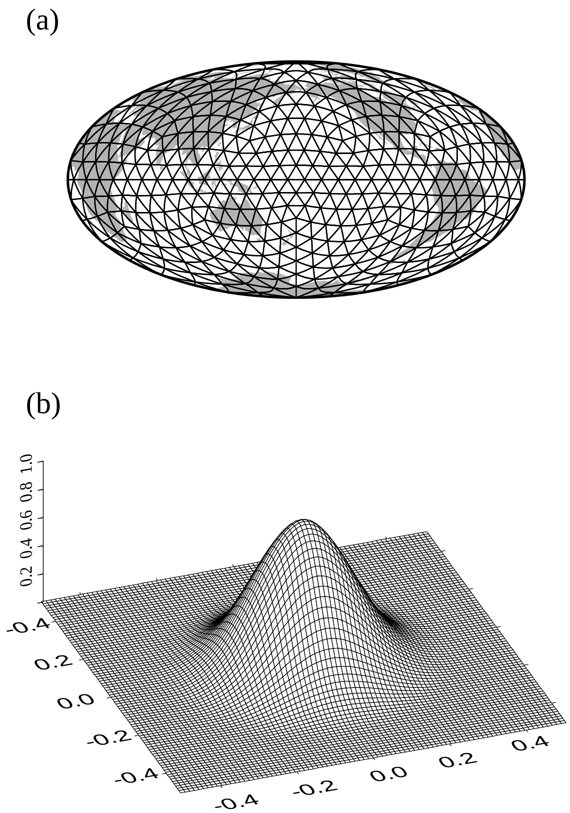

Figure 3: (a) Horizontal parameterization of the global model. The Earth's surface is divided into 362 equal-area triangles. Each knot location corresponds to the center of a spherical B-spline. (b) A 3-D view of a spherical B-spline used in the lateral expansion. The spline function is localized, axially symmetric, and has been normalized to 1.

It is therefore important to use data in tomographic inversions which have maximum sensitivity to upper mantle structure. Figure 2 shows the average value of the diagonal elements of an inner product matrix used in an inversion for S wave velocity [*Gu et al.*, 2000]. Maximum sensitivity to the upper mantle is provided by the use of surface wave dispersion measurements and body and surface waveforms with periods longer than 45 s. Differential and direct travel times are mostly sensitive to lower mantle structure. For a P wave inversion Figure 2 would be similar, without the Love wave measurements and with the travel times for corresponding P phases substituted. In deriving our new global mantle model, we have therefore made use of a variety of datasets.

For the model parameterization we choose a combination of spherical and radial B-splines. Horizontally, the spherical splines are centered at 362 uniformly distributed knots (Figure 3). The number of free parameters is close to that involved in a degree-18 spherical harmonic expansion. Perturbations to PREM in both P and S velocity are represented by

$$\frac{\delta v(r,\theta,\phi)}{v_\circ} = \sum_{i,j} C_{ij} S_j(\theta,\phi) \cdot B_i(r) \tag{1}$$

where r, θ, and ϕ are radius, colatitude, and longitude, respectively, S_j is the jth spherical spline, and B_i the ith radial spline. C_{ij} are the model coefficients to be determined. The mathematical form of the S_j is given in *Wang and Dahlen* [1995]. A 3-D view of one of the spherical spline functions is shown in Figure 3b. The maximum amplitude is localized to the vicinity of the central knot. The spline approach thus combines some of the advantages of using a local, block parameterization with the advantage of producing a smooth model.

Radially, the parameterization consists of 14 cubic B-splines distributed non-uniformly with depth (Figure 4). The local support of B-splines allows for fast computation since only neighboring splines are needed to compute the value of the model at a given point. The second derivatives of the splines vanish at both ends. We use the split parameterization of *Gu et al.* [2000] in which the radial splines are split at the boundary between the upper and lower mantle (670 km) in order to detect possible sharp changes in velocity variations across this depth. Six B-splines are used for the upper mantle portion and 8 for the lower mantle.

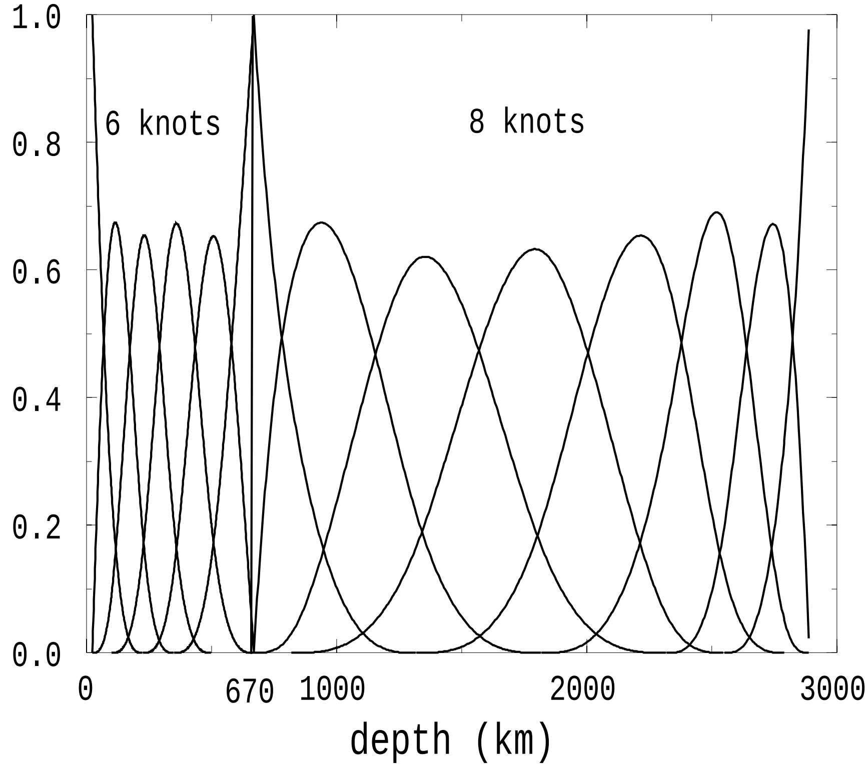

Figure 4: Split radial B-splines used for parameterization of the global model. Two sets of splines are used: 6 for the upper mantle and 8 for the lower mantle. This parameterization provides higher nominal resolution near the 670-km discontinuity.

We are currently using a large of number of datasets in the inversions as well as developing others for future use. The compressional velocity model is determined by travel times of direct P and core phases (PKP, PcP) to improve resolution in the lowermost mantle. These datasets were adopted from those of *Engdahl et al.* [1998] but have been improved through relocation of the earthquakes in a 3-D model (SP12). The travel times were then formed into summary rays with sources at the center of 2° x 2° cells. Events deeper than 50 km were grouped by depth into bins with a thickness of 100 km. Events whose epicenter moved more than 1° from the location published by *Engdahl et al.* [1998] were discarded. The total number of summary rays is 626,073 for P, 215,590 for PKP, and 68,473 for PcP. We also make use of the Love and Rayleigh wave dispersion measurements published by *Ekström et al.* [1997] in the period range 35-150 s, which have their maximum sensitivity in the upper 200-300 km of the mantle. These were previously used by *Ekström and Dziewonski* [1998] to derive a 3-D model of mantle shear wave velocity expanded in spherical harmonics up to degree 20 as part of an earlier phase of this project. For shear velocity inversions, we also use the direct and differential travel time datasets described in *Su et al.* [1994]. They consist of approximately 45,000 measurements of S, SS, ScS, SS - S, S - SKS, ScS - S, and $SKKS$ - SKS travel times. Future models will also include the extensive dataset of body and surface waveforms compiled over the years at Harvard.

We solve the inverse problem by non-linear, iterative least-squares analysis. The equations are of the form

$$\delta \mathbf{d}_0 = \mathbf{A}\delta\mathbf{x} \tag{2}$$

where $\delta\mathbf{x}$ is the perturbation in the model from the previous iteration, $\delta\mathbf{d}_0$ is the corresponding unexplained portion of the data vector, and $\mathbf{A}$ is the kernel matrix. For the final iteration, we require that

$$|\mathbf{A}\delta\mathbf{x} - \delta\mathbf{d}_0|^2 + \lambda^2|\delta\mathbf{x}|^2 + \eta^2 g^2 = min \tag{3}$$

where λ and η are damping factors and g^2 is the integrated squared gradient of the model:

$$g^2 = \int_\Omega \int_{r_{cmb}}^{r_{moho}} |\bigtriangledown(\delta v/v_0)|^2 dr d\Omega. \tag{4}$$

The least-squares model solution at each iteration is computed by the method of Cholesky factorization [*Trefethen and Bau*, 1997].

An initial P wave velocity model is shown in Figure 5. With respect to model BDP98, the new model shows higher amplitude velocity perturbations in the upper mantle, which may be the result of the 3-D relocation of the sources prior to inversion. Use of the surface wave measurements produces lower velocities throughout the eastern and central Pacific than are observed using travel times alone. At the bottom of the mantle, the new model has slightly lower amplitude than BDP98 and much lower amplitude than model SP12, especially in the southern hemisphere. This may be partly due to the relatively coarse separation of the radial splines in the lower mantle in this parameterization. Since, as noted above, the upper mantle is most critical for accurate earthquake location, the new model may improve upon the location accuracy provided by BDP98 and other high-resolution models. Our new models will be tested using the dataset of calibration events compiled at Harvard and other institutions.

Constraining earthquake locations on mid-ocean ridges and transforms

In addition to the construction of more detailed and accurate global velocity models, we have focused our efforts on building a larger database of reference or "ground-truth" events to be used for model calibration. So far, the geographical distribution of events with locations known to an accuracy of 5 km or better is extremely limited. Many of these events are nuclear explosions and are concentrated in only a few source regions, making comprehensive testing of velocity models and calibration of new seismic stations very difficult. We are experimenting with relocation of events on remote plate boundaries using a technique which constrains the location to a point along the plate boundary. For example, by limiting the process to only large events for which a Centroid Moment Tensor (CMT) solution exists, an assessment can be made as to whether an event should be located along a mid-ocean ridge segment or an adjacent transform fault based on its focal mechanism. We then perform a transformation of coordinates into the system defined by the pole of rotation between the two plates, such that the ridge or transform fault lies along a parallel or meridian. We use accurate bathymetry to determine the precise location and trend of the boundary [*Smith and Sandwell*, 1997]. In the subsequent location process, the coordinate (latitude or longitude) which corresponds to the plate boundary is held fixed. In this way, we are able to develop a master set of events which can then be used to relocate other smaller events in the same region.

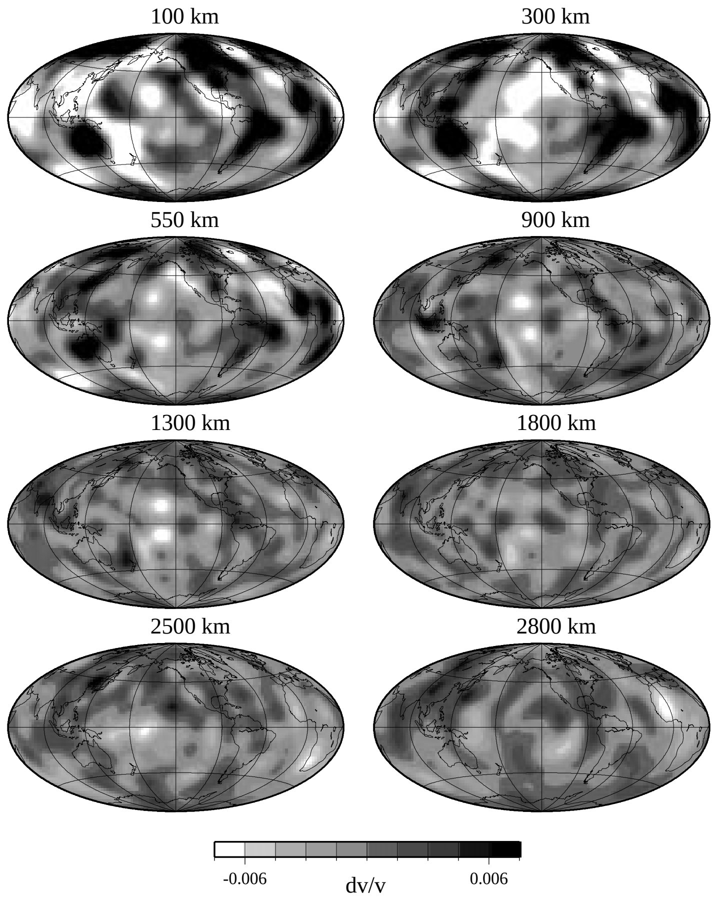

Figure 5: Model of P wave velocity at various depths obtained in joint inversion of compressional and shear velocity using travel times and surface wave phase velocities. Scale shows fractional velocity perturbation from PREM.

We have thus far compiled a catalog of these events along most of the transform faults in the Atlantic, Pacific, and Indian oceans. Figure 6 shows the relocation of events along the Romanche Fracture Zone (RFZ) in the central Atlantic. The top panel shows the ISC locations of events in the CMT catalog. After relocation (middle panel), the strike-slip earthquakes associated with the RFZ lie along the trend defined by the bathymetry. The ISC locations are systematically displaced to the south relative to the RFZ, and the apparent mislocations are as large as 40 km. The results obtained thus far are promising but require extensive testing to determine the accuracy of the relocations.

Conclusions and Recommendations

Although the current generation of global velocity models provides substantial improvement over 1-D velocity models for teleseismic earthquake location [*Smith and Ekström*, 1996; *Antolik et al.*, 2000], there is still much room for improvement. The consistent accurate location of small events to within the area specified for on-site inspection under the CTBT (1000 km^2) will likely require the use of regional phases in addition to teleseismic travel times, and also possibly consideration of anisotropy or arrival angle data. To this end, we are developing new global models which combine a local parameterization with the advantages

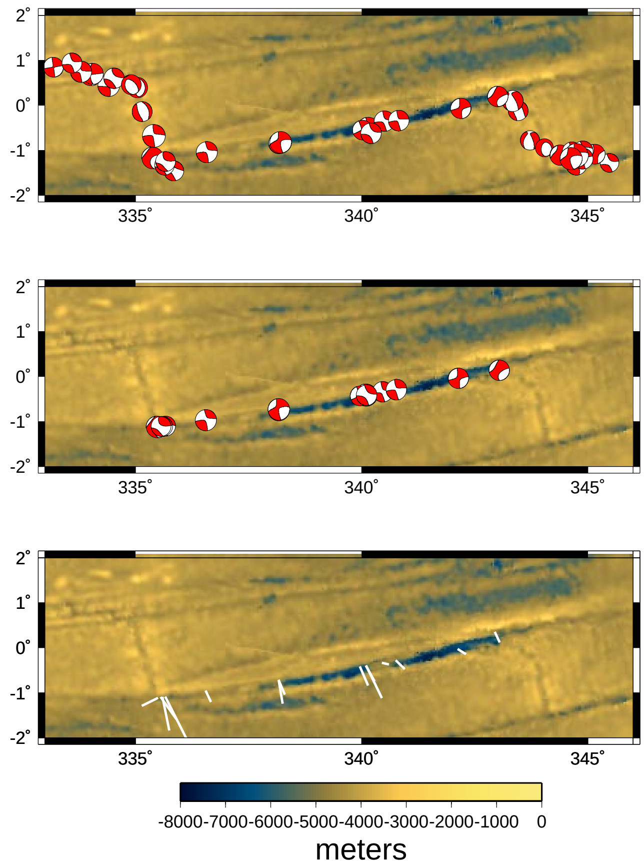

Figure 6: Relocation of earthquakes along the Romanche Fracture Zone in the central Atlantic. The top panel shows the focal mechanisms of large earthquakes in the region plotted at their ISC locations. After identification of the strike-slip events to be constrained to the RFZ, these events are relocated after coordinate transformation (middle panel). The bottom panel shows the change in location of these same events, multiplied by a factor of 2.5. Bathymetry is taken from *Smith and Sandwell* [1997].

of smoothness over large distances. This will enable straightforward replacement of portions of the global model by a more detailed model covering a particular region. The smooth parameterization will allow simple computation of 3-D raypaths.

Testing of new global and regional velocity models, as well as calibration of newer IMS stations, will benefit from the compilation of reference events in remote regions. Our database of relocated events on mid-ocean plate boundaries will also enable construction of more accurate travel time and phase velocity datasets.

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